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Separation of Fuzzy Topological Space

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ABSTRACT
In this paper I discussed about some separation properties of fuzzy topological space. The definition of fuzzy compactness does not hold good on the definition of fuzzy Hausdorff space, we introduce a new definition of proper compactness and prove some interesting results related with it.

Keywords
Fuzzy set, Fuzzy topological space, Fuzzy Hausdorff space, compactness, T2- space.

1. INTRODUCTION
Let X be any set. A fuzzy set A in X is characterized by membership function $\mu_A : X \to [0,1]$ A fuzzy singleton p in X is a fuzzy set with the membership function μ_p defined by $\mu_p(X) = y \text{ if } x = x_0$ = 0 otherwise

where $y \in (0,1). x_0$ fuzzy set in X, if $\mu_p(x_0) < \mu_A(x_0)$

Hence $p \notin A \text{iff } \mu_p(x_0) \geq \mu_A(x_0)$

Definition 1 :
A fuzzy topological space is said to be Housdorff of fuzzy T2 if the following conditions are satisfied :

For any $p, q \notin X$,

(i) If $x_p \neq x_q$ then there exist open sets and Gp and Gq, $p \in \bar{G}_p \text{ and } q \in G_q, p \in \bar{G}_p$

(ii) xp = xq and $\mu_p(x_p) < \mu_q(x_q) \Rightarrow$ there exist an open Gp with $p \in G_p \text{ but } q \notin \bar{G}_p$.

By Srivastava & Lal [8].

Definition 2 :
A fuzzy set A is said to be open for each , there exists an open set G with $p \in G \subseteq A$. Out definition of open set is equivalent to that given by Change [1].

Definition 3 :
A fuzzy topological space is said to be a fuzzy T_1- space if the singletons are closed. By Hutton & Reilly [2].

2. THEOREM (I)
Every fuzzy T_2- space is a fuzzy T_1- space.

Proof : Let (X, T) be a fuzzy T_2- space.

Case (i) : Let p be a fuzzy point in X, and let $q \in \{p\}'$ be arbitrary. Then, there exist an open set G_q containing q such that

$\mu_{(p)}(x_p) \geq \mu G_q(x_p)$

Hence G_q {p}. So, {p} being the union of open sets, is an open set. This implies that {p} is closed.

Case (ii) : Let p be a crisp point and converges to zero, then we can find a sequence of open sets $\{G_{pq_n}\}_{n \in N}$ with $p \in G_{pq_n}$ and $q_n \bar{G}_{pq_n}$ for all n, as (X, T) is fuzzy T_2.

If $P = \cap \bar{G}_{pq_n}$ then p is a closed set with $\mu_p(x_q) = 0 \text{ and } \mu_p(x_p) = 1$

$p \in V_{pq_n}$

So, P' is an open set with $q \in P' \{p\}'$ Consequently, $\{p\}'$ is an open set i.e. $\{p\}$ is closed. So, (X, T) is fuzzy T_1-space.

We now introduce the concept of properly compact space in a fuzzy

topological space and prove an analogous result of general topology which holds for a compact Hausdroff space.By Palaniappan[3] & Zadeh[4]

Definition 4 :
A family $\{G_1 : i \in I\}$ open sets in fuzzy topological space X is called a proper open cover of a fuzzy set A in X if for every $x \in X$ there exists a member G_{i_x} of this family such that of this family such that $\mu G_{i_x}(x) \geq \mu_A(x)$

The family $\{G_i : i \in I\}$ is called a proper open sub cover of $\{G_i : i \in I\}$ if it is itself a proper open cover of A.
By Lowen[5].

Definition 5 :
A fuzzy set A in a fuzzy topological space X is said to be properly compact if every proper open cover of A is reducible to a finite proper open sub cover.

By Lowen [6] & Rodabaugh[7].

3. THEOREM 2
Every properly compact set in a fuzzy T_2- space is closed.

Proof : Let A be a properly compact set in a fuzzy T_2-space X. We choose a point with

$$\mu_p(x_p) > \mu_A(x_p) \quad \text{(i)}$$

since X is T_2, there exists an open G_p with

$$\mu_A(x_p) < \mu_{G_p}(x_p) \quad \text{(ii)}$$

and $$\mu_p(x_p) \geq \mu_{G_p} - (x_p) \quad \text{(iii)}$$

Thus, for each such $\in X$ there exists a family $\{G_{pq} : x_q \in X\}$ of open sets

with $\mu_A(x_q) < \mu_{G_{pq}}(x_q)$ for all $x_q \in X$ there exists a family $\{G_{pq} : x \in X\}$ of open sets $\mu_A(x_q) < \mu_{G_{pq}}(x_q)$ with (iv)

So, $$\mu_A(x_q) \leq \sup x_q \in \mu_{G_{pq}}(x_p)$$

This implies that $$A \subseteq \cup_{x_q \in X} G_{pq}$$

Thus, the family $\{G_{pq} : x_q \in X\}$ is a proper cover of A. Since A is properly compact, there exists a finite subfamily, say,

$\{G_{pq1}, G_{pq2}, \ldots\ldots, G_{pqm}\}$ of $\{G_{pq} : xq \in X\}$ of $\{G_{pq} : x_q X\}$ with

$$A \subset \cup G_{pqk}. \text{Then} A \subset \bigcup_{K=1}^{m} \bar{G}_{pqk} = F_p \text{ (suppose)}$$

Then F_p is closed and $\mu_p(x_p) \leq \mu_{Fp}(x_p)$ for all $x_q \in X$ (v)

We also have $\mu_p(x_p) \geq \mu_{Fp}(x_p)$ (iv)

Then, for all satisfying the condition (i), the family $\{F_p\}$ where F'_p is open in X, satisfies the conditions (v) & (vi). Consequently we have $\mu_A(x_q) = \operatorname{Inf}_p \mu_{Fp}(x_q)$ for all $x_q \in X$.

Then, $A = \bigcap_p F_p$ with establishes that A is closed.

4. CONCLUSION
From the above theorems and definitions it is clear that A fuzzy set in a fuzzy topological space is said to be properly compact if every proper open cover of a fuzzy set is reducible to a finite proper open sub cover and every properly compact set in a fuzzy T_2-space is closed. Thus the definition of fuzzy compactness does not hold good on the definition of fuzzy Hausdorff space.

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