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Numerical Modeling of the Role of Reverse Parameter in a range of Population Inversions of Differential rate Equations of Tm-doped Material

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ABSTRACT
This paper numerical investigates the impact of reverse cross relaxation parameter on the pump efficiency using differential geometry of rate equations of thulium-doped (Tm-doped) tellurite material by examine certain reverse cross-relaxation processes, relating to series of population inversions. The primary aim of this research is to obtain a set of data that assists in predicting material performance in relation to a number of instances of the reverse cross-relaxation parameter to determine the optimum value of reverse cross- relaxation parameter. It is pointed out that a deep examination of the reverse cross-relaxation process ($^3F_4$,$^3F_4$,→$^3H_6$,$^3H_4$) was not available with the available data. The study demonstrates that reverse cross-relaxation may influence the pump efficiency of Tm-doped tellurite material. Therefore the research provides a set of parameters enabling evaluation of the reverse cross-relaxation process, thus illustrating how this process affects pump efficiency during a range of population inversions within differential geometry principle (0-50% in this study). The findings indicate that appropriate measurements of reverse cross-relaxation parameter may have a considerable effect on simulations for laser and amplifier devices. Moreover, the research shows that measurements of reverse cross-relaxation parameter for different glass types can facilitate identification of an appropriate host for various applications.

General Terms
Reverse cross relaxation parameter, Cross relaxation, Thulium doped material. Differential rate equations

Keywords
Differential rate equations, Thulium-doped material, Pumping efficiency, Pump power, Reverse cross relaxation parameter, Cross-relaxation process, Population inversion

1. INTRODUCTION
The emission spectrum of thulium (Tm^{3+}) is broad, being approximately 1.8 microns. This means that Tm^{3+} is a crucial component of many infrared laser applications, such as precision cutting, the surgical removal of body tissues, and sensing [1-6]. Moreover, Tm^{3+} is characterized by the cross-relaxation process ($^3H_4$, $^3H_6$ → $^3F_4$,$^3F_4$), during which two ions move into the laser's upper level via each single pumping photon. This cross-relaxation process results in improved pumping quantum efficiency, with lasing at 1.8 µm. However, most laser simulations cannot calculate or cannot take account of the reverse cross-relaxation process ($^3F_4$,$^3F_4$→$^3H_6$,$^3H_4$), which is important because it reduces the efficiency of the cross-relaxation process.

Previous literatures show that laser host materials utilize various sorts of glass; for example, tellurite, silica, germanate, and fluoride [7-11]. Amongst of all the oxide glasses, tellurite glasses have the lowest range of photon energies (approximately 750 cm^{-1}) and high rare earth ion solubility. Independently from the type of glass, the design process of lasers involves precision modeling, together with the comparative analysis of different doping and inversion values[12],[13]. Although there is considerable evidence regarding lifetimes and cross-sections, the parameters of the ion-ion interactions (i.e. the reverse transfer process) [12-17] are more difficult to determine. In particular, these usually are not available as parameters validated over a large interval of doping level and therefore unsuitable for doping level optimization. A number of previous researchers, such as [18],[19], have concentrated on measuring the reverse transfer process in different samples of Tm-doped tellurite glass, comparing it with experimental data via precise calculations and best set-up validated by fitting experimental fluorescence decay curves of both $^3H_4$ and $^3F_4$ levels. Those researchers demonstrated that a series of samples of Tm-doped tellurite glass can be fit with doping level variations by a factor of 30 (0.36-10 mol%), the same parameters are applied to each sample. Those studies also showed that the reverse cross-relaxation process parameter can be calculated when the slow decaying fluorescence tails are fitted. Of particular interest is the fact that those researchers employed different methodology to that deriving from Kushida as in Ref [20]. The methodology refers to *a* as the ratio between reverse and cross relaxation processes and identified its value as 0.03 ($P_{22} = a \times P_{41}$), i.e. *a* equaled 3% in Ref. [18]. Taking into account these studies, this paper evaluate and assess the impact of the reverse cross-relaxation parameter on the required pump power to reach a given population inversion. The findings indicate that selection of suitable glass type depends on the value of the reverse cross relaxation process.

2. THEORETICAL MODELING
This research draws on the energy levels scheme and the transitions given in fig. 1 [13],[21].

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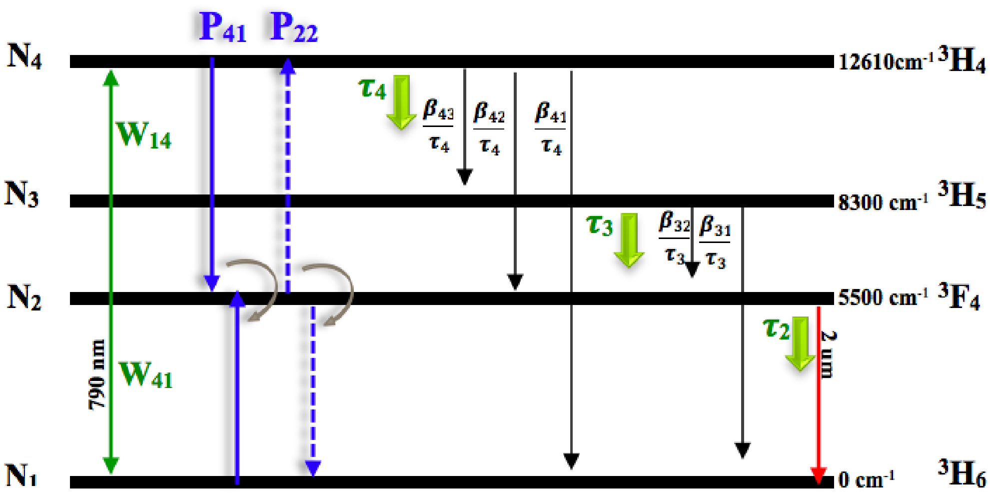

Fig. 1: Energy Level Scheme of Thulium

This figure illustrates the four lowest energy levels of the Tm^{3+} ion, including laser and pump transitions, direct and reverse cross-relaxation processes and spontaneous decay paths. The relevant differential rate equations are as follows:

$$\frac{dN_4}{dt} = W_{14}N_1 - W_{41}N_4 - \frac{N_4}{\tau_4} - P_{41}N_4N_1 + P_{22}N_2^2 \quad (1)$$

$$\frac{dN_3}{dt} = -\frac{N_3}{\tau_3} + \frac{\beta_{43}N_4}{\tau_4} \quad (2)$$

$$\frac{dN_2}{dt} = 2P_{41}N_4N_1 - 2P_{22}N_2^2 - \frac{N_2}{\tau_2} + \frac{\beta_{42}N_4}{\tau_4} + \frac{\beta_{32}N_3}{\tau_3} \quad (3)$$

$$\frac{dN_1}{dt} = -W_{14}N_1 + W_{41}N_4 + P_{22}N_2^2 - P_{41}N_4N_1 + \frac{N_2}{\tau_2} + \frac{\beta_{41}N_4}{\tau_4} + \frac{\beta_{31}N_3}{\tau_3} \quad (4)$$

where N_1 ,N_2 ,N_3 and N_4 are the population of the energy levels $^3H_6$ (ground level), $^3F_4$ (upper laser level), $^3H_5$ and $^3H_4$ (pump level), respectively; W_{14} ,W_{41} are the pump rates, τ_i the lifetime of the i-level, and β_{ij} are branch ratios from the *i*-to J-level [17]. The coefficients P_{ij}describe the energy transfer processes: P_{41} ($^3H_4$, $^3H_6 \rightarrow$ $^3F_4$, $^3F_4$) is the cross relaxation constant, which is proportional to doping level [14],[17], and P_{22} ($^3F_4$,$^3F_4$,$\rightarrow$$^3H_6$,$^3H_4$) is the reverse cross-relaxation process constant, the investigation of which is the main aim of this study.

3. SIMULATION

This model was utilized to determine and evaluate the reverse cross relaxation parameter of a sample of Tellurite glass, with doping level equal 4 mol%. The relevant values are shown in Table 1. The pump cross-section (at 790 nm) was taken from Ref. [17] while the emission cross-section was calculated using Ref [22].

Although it was possible to calculate many parameters, or simply to extract relevant data from the literature such as lifetime values, cross-relaxation constants or branching ratios, the reverse cross-relaxation process had to be measured in order to complete the set of parameters. This also allowed the usage of Eq.1-4 in simulating the Tm-doped laser over a broad spectrum of concentrations, and subsequently to examine the effect of reverse cross-relaxation parameter on the level of pumping required to attain a given population inversion.

Table 1:List of parameters used in the modeling

Parameters	Symb	values		Ref.
Coefficient of Cross Relaxation	Cr	$1.81*10^{-23}$ m^3 s^1 mol^{-1}		[14]
Pump wavelength	λ_p	790 nm		
Cross Relaxation	P_{41}	Mole *Cr (m^3 s^{-1})		[14]
Reverse of Cross relaxation	P_{22}	$0.03*P_{41}$		[18]
Absorption pump cross-section	σ_{ap}	$8*10^{-25}$ m^2		[17]
Emission pump cross-section	σ_{ep}	$2.2*10^{-25}$ m^2		[22] *
Pump Intensity	I_p	$1.3*10^3$ W/cm^2		
Branch ratio	β_{41}	$^3H_4 \rightarrow {}^3H_6$	0.903	[17]
	β_{42}	$^3H_4 \rightarrow {}^3F_4$	0.076	[17]
	β_{43}	$^3H_4 \rightarrow {}^3H_5$	0.02	[17]
	β_{31}	$^3H_5 \rightarrow {}^3H_6$	0.979	[17]
	β_{32}	$^3H_5 \rightarrow {}^3F_4$	0.020	[17]

*Calculated by McCumber equation from [22]

4. RESULT AND DISCUSSION

The required level of pump intensity to reach a given population inversion was determined in order to assess the impact of reverse cross-relaxation on pumping efficiency and device performance. This should be considered an estimate, however, because other factors may well interact with the reverse cross-relaxation process during laser or amplifier operations. As Ref. [23-27], demonstrated that these factors include photodarkening, since pathways involve population of Tm levels affected by reverse cross-relaxation. Fig. 2 shows, for sample T4, the population inversions from 0 to 50% , where the inversion represents the difference between N_2 and N_1 normalized to the doping concentration ((i.e., $(N_2-N_1)/N_t$).

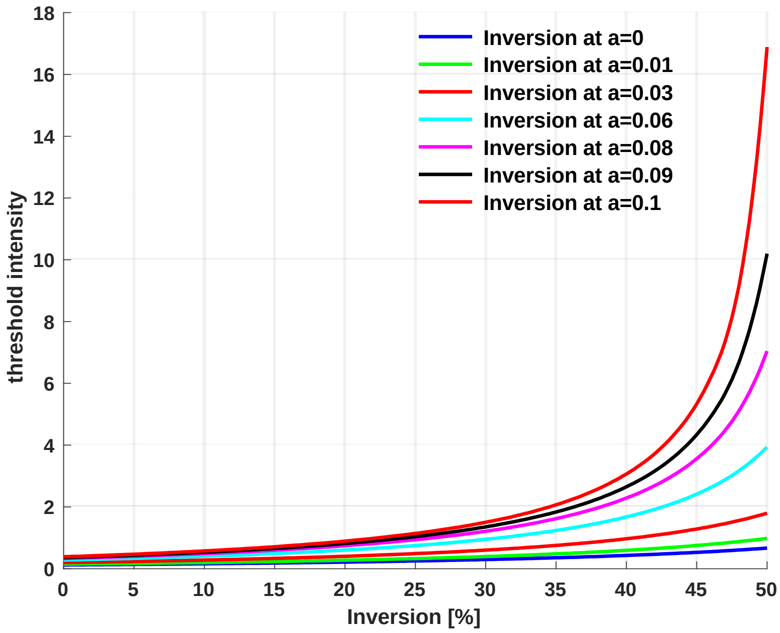

Figure 2: Threshold intensity versus parameter a for inversion from 0% to 50%. Calculations were done for sample T4

At this stage of the research, from Fig. 2, it was observed that the dependence was not linear, and at high concentrations, the dependence becomes increasingly quadratic. This is demonstrated in Eq.1, in which P_{22} (reverse cross relaxation parameter) is measured by *a*, and has a quadratic dependence on N_2, and the dependence on the pump power is linear. So the equation balance may justify such a quadratic dependence. Interestingly, when the value of *a* is underestimated by a factor of 3, the required pump intensity is significantly underestimated, which concurs with Ref. [18]. Finally it was noted that a low value of a may be of considerable benefits in the case of higher inversion (for instance, in relation to amplifier devices). Thus it seems an appropriate conclusion is that a choice of the most appropriate glass may depend on the specific application and should consider a whole set of parameters, including reverse cross-relaxation. Fig. 2 also indicates that the effect of reverse cross-relaxation on pump efficiency is larger in case of a higher population inversion (in relation to amplifier devices, for example). This also supports the conclusion that application may influence glass selection. In line with this, Jackson and King [12] showed that silica glass has a longer $^3H_4$ lifetime (30%) at the doping concentration of 5.75 mol%, but a = 0.08, while Fagundes-Peters, Camargo, and Nunes.[28] utilized Kushida's model [20] to calculate *a* value lower than 0.01 in fluoroindogallate glass. From these results it may be concluded that determining the reverse-cross relaxation value for various glass types might offer a useful means of guiding researchers to identify of appropriate glass hosts for all applications.

5. CONCLUSION

This paper demonstrates the impact of the reverse cross relaxation parameter on pump efficacy under a range of population inversions by using differential geometry in rate equations for Tm-doped material. It have been able to identify a set of spectroscopic parameters for Tm-doped tellurite glass, allowing calculating the reverse cross relaxation values over a wide range of doping levels within differential rate equations. This was realized by determining all applicable parameters including the reverse cross-relaxation constant. Therefore this research has provided a means of assessing the effect of reverse cross-relaxation parameter on pump efficiency during various population inversions. Most importantly, this method may be used with all glass varieties. Although a low inversion regime is most favorable for the cross-relaxation parameter, cross sections, branching ratios, and lifetime measurements, a further series of measurements, utilizing increased pump level, should be conducted to accurately define the reverse cross-relaxation parameter. Therefore, with the principal research objective accomplished, the authers are confident in stating that suitable calculations of the reverse cross-relaxation parameter are likely to have a considerable impact on the simulation.

This set will allow researchers to appropriately simulate active device and to find the optimum doping level. This study also will enable scientists to determine the appropriate glass for the desired application. It also will enable researchers to study and predict the behavior of devices accurately, where this study take into account the whole set of parameters, including reverse cross-relaxation parameter.

6. ACKNOWLEDGMENTS

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