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Total Edge Dominating Functions of Corona Product Graph of a Cycle with a Complete Graph

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ABSTRACT
Graph theory is one of the most flourishing branches of modern mathematics and computer science. Domination in graphs has been studied extensively in recent years and it is an important branch of graph theory. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [11, 12].

In this paper some results on minimal total edge dominating sets and minimal total edge dominating functions of corona product graph of cycle with a complete graph are presented.

Keywords
Corona Product, Total edge dominating set, Total edge domination number.

Subject Classification: 68R101.

1. INTRODUCTION
Domination Theory has a wide range of applications to many fields like Engineering, Communication Networks, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[1], Cockayne, E.J. and Hedetniemi, S.T. [6] have studied various domination parameters of graphs.

Products are often viewed as a convenient language with which one can describe structures, but they are increasingly being applied in more substantial ways. Every branch of mathematics employs some notion of product that enables the combination or decomposition of its elemental structures. Frucht and Harary [10] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$.

The concept of edge domination was introduced by Mitchell and Hedetniemi [16] and it is explored by many researchers. Arumugam and Velammal [5] have discussed the edge domination in graphs while the fractional edge domination in graphs is discussed in Arumugam and Jerry [4]. The complementary edge domination in graphs is studied by Kulli and Soner [15] while Jayaram [14] has studied the line dominating sets and obtained bounds for the line domination number.

The bipartite graphs with equal edge domination number and maximum matching cardinality are characterized by Dutton and Klostermeyer [9] while Yannakakis and Gavril [17] have shown that edge dominating set problem is NP complete even when restricted to planar or bipartite graphs of maximum degree. The edge domination in graphs of cubes is studied by Zelinka [18].

CORONA PRODUCT OF C_n AND K_m
The corona product of a cycle C_n with a complete graph K_m is a graph obtained by taking one copy of a n – vertex graph C_n and n copies of K_m and then joining the i^{th} vertex of C_n to every vertex of i^{th} copy of K_m . This graph is denoted by $C_n \odot K_m$.

The vertices of C_n are denoted by $v_1, v_2, \dots, v_n$. The edges in C_n are denoted by $e_1, e_2, \dots, e_n$ where e_i is the edge joining the vertices v_i and v_{i+1}, $i \neq n$. For $i = n$, e_n is the edge joining the vertices v_n and v_1.

The vertices in the i^{th} copy of K_m are denoted by $w_{i1}, w_{i2}, \dots, w_{im}$. The edges in the i^{th} copy of K_m are denoted by $l_{ij}, j = 1,2, \dots, \frac{m(m-1)}{2}$.

There are another type of edges in G denoted by $h_{ij}, i = 1,2, \dots, n$ and $j = 1,2, \dots, m$ is the edge joining the vertex v_i of C_n to vertex w_{ij} of i^{th} copy of K_m. These edges which are in G and related to the i^{th} copy of K_m are denoted by $h_{i1}, h_{i2}, \dots, h_{im}$ and these are adjacent to each other and incident with the vertex v_i of C_n.

Some properties of corona product graph $G = C_n \odot K_m$ are studied by Anita [2] and some results on minimal edge dominating sets and functions of this graph are presented in [3].

2. TOTAL EDGE DOMINATING SETS AND TOTAL EDGE DOMINATING FUNCTIONS
The concepts of total dominating functions and minimal dominating functions are introduced by Cockayne et al. [7,8] and Henning [13]. In this section, we introduce the concept of total edge dominating functions and minimal total edge dominating functions. Further, we prove some results related to total edge dominating functions of graph $= C_n \odot K_m$. First let us recall some definitions.

Definition: Let $G(V, E)$ be a graph without isolated edges. A subset T of E is called a **total edge dominating set** (TEDS) if every edge in E is adjacent to at least one edge in T.

If no proper subset of T is a total edge dominating set, then T is called a **minimal total edge dominating set** (MTEDS) of G.

Definition: The minimum cardinality of a MTEDS of G is called a **total edge domination number** of G and is denoted by $\gamma'_t(G)$.

Definition: Let $G(V,E)$ be a graph. A function $f:E \to [0,1]$ is called a **total edge dominating function** (TEDF) of G if $f(N(e)) = \sum_{e' \in N(e)} f(e') \geq 1$, for each $e \in E$.

Definition: Let f and g be functions from E to $[0,1]$. We define $f < g$ if $f(e) \leq g(e)$ for all $e \in E$, with strict inequality holds for at least one edge $e \in E$.

A TEDF f of G is called a **minimal total edge dominating function** (MTEDF) if for all $g < f$, g is not a TEDF.

We need the following Theorem which is presented in [2].

Theorem 2.1: The adjacency of an edge e in $G = C_n \odot K_m$ is given by $adj(e) = \begin{cases} 2m+2, \text{if } e = e_i \in C_n, \\ 2m-2, \text{if } e = l_{ij} \in i^{th} \text{ copy of } K_m, \\ 2m, \text{if } e = h_{ij} \in G = C_n \odot K_m. \end{cases}$

Theorem 2.2: The total edge domination number of $G = C_n \odot K_m$ is $n(m-1)$.

Proof: Let $G = C_n \odot K_m$ be the given corona product graph of C_n with K_m.

We know that the vertex set V of G i

$V = \{v_1, v_2, \dots, v_n; w_{i1}, w_{i2}, w_{i3}, \dots, w_{im};\ i = 1,2,3,\dots,n\}$ and $E = \{e_1, e_2, \dots, e_n; h_{i1}, h_{i2}, \dots, h_{im}; l_{i1}, l_{i2}, \dots, l_{ik}, i =$ *1,2,3,...,n;k=1,2,...,m(m−1)2.*

The vertices $v_1, v_2, \dots, v_n$ are vertices of C_n and $w_{i1}, w_{i2}, w_{i3}, \dots, w_{im}$ are vertices in the i^{th} copy of K_m. Further e_i *is* the edge joining the vertices v_i and $v_{i+1,}$ if $i \neq n$ and e_i is the edge joining the vertices v_i and v_1 if $i = n$. The edge h_{ij} is the edge joining the vertices v_i and w_{ij}. The edge l_{ij} is the edge joining the vertices w_{ij} and $w_{i(j+1)}$.

Let G_i denote the vertex induced sub graph on the vertex set $V_i = \{v_i, w_{i1}, w_{i2}, \dots, w_{im}\}$.

The edges in G_i are h_{ij} and l_{ij} where $j = 1, 2, \dots, m$.

Let $T_i = \{h_{i1}, l_{ij}; j = 1,2,\dots,(m-2)\}$ be the set of edges in G_i, $i = 1,2,3,\dots,n$.

Here l_{ij}, $j = 1,2,3,\dots,(m-2)$ are the edges in the outer Hamilton cycle of K_m. We can see that these edges dominate all the remaining edges in the i^{th} copy of K_m. The edge h_{i1} dominates the edges e_i, e_{i+1} of C_n and the edges h_{ij}, $j = 2, 3, \dots, m$ of G_i.

Further h_{i1} and l_{i1} are adjacent.

Thus $T_i = \{h_{i1}, l_{ij}; j = 1,2,\dots,(m-2)\}$ becomes a total edge dominating set in G_i.

Let $T_1, T_2, \dots, T_n$ be the total edge dominating sets in $G_1, G_2, \dots, G_n$ respectively.

Let $T = T_1 \cup T_2 \cup \dots \cup T_n$.

Then obviously T becomes a total edge dominating set of G.

We claim that T is a minimal total edge dominating set of G.

Suppose we delete the edge h_{i1} from T_i. Then obviously all the edges h_{ij}, $j = 1,2,,m$ are not dominated by the edges in $T_i - \{h_{i1}\}$.

Similarly if we delete any edge l_{ij} for $j = 1,2,\dots(m-2)$ say l_{ik} from T_i, then all the edges in K_m are not dominated by $T_i - \{l_{ik}\}$ in the i^{th} copy of K_m, since the selection of $(m-2)$ edges l_{ij} in the outer Hamilton cycle of K_m is the minimum number.

Therefore T_i is a minimal total edge dominating set in G_i. As $(m-2)$ edges l_{ij} and one edge h_{i1} are selected from G_i into T_i and there are n such G_is, it follows that the cardinality of T is $n(m-1)$.

Therefore $\gamma'_t(C_n \odot K_m) = n(m-1)$.

Theorem 2.3: Let T be a minimal total edge dominating set of $G = C_n \odot K_m$. Then a function $f: E \to [0,1]$ defined by

$$f(e) = \begin{cases} 1, & \text{if } e \in T, \\ 0, & \text{otherwise.} \end{cases}$$

becomes a minimal total edge dominating function of $G = C_n \odot K_m$.

Proof: Let $G = C_n \odot K_m$ be the given graph. Then as per Theorem 2.2 we know that $T = \{h_{i1}, l_{ij}, i = 1,2,3,\dots,n, j =$ *1,2,3,..., (m−2)* is a minimal total edge dominating set of *G*.

Now the following cases arise.

Case 1: Let $e_i \in C_n, i = 1,2,\dots,n$ be such that $adj(e_i) = 2m+2$. Then $N(e_i)$ contains two edges of C_n and $2m$ edges which are drawn from the vertices v_i and v_{i+1} respectively to the m vertices of i^{th} and $(i+1)^{th}$ copies of K_m.

$$\text{Therefore} \sum_{e \in N(e_i)} f(e) = 0 + 0 + \underbrace{[0 + 0 + \cdots + 0]}_{(2m-2)-times} + 1 + 1 = 2.$$

Case 2: Let $l_{ij} \in i^{th}$ copy of K_m be such that $adj(l_{ij}) = 2m-2$.

We see that $(m-2)$ edges l_{ij} are taken into the total edge dominating set T from each G_i and functional value 1 is assigned to these edges and these edges are taken on the outer Hamilton cycle of K_m continuously .

Sub case (i): Suppose $f(l_{ij}) = 1, j = 1$ and $m = 3$. That is $l_{ij} \in$ T. $N(l_{i1})$ contains an edge h_{i1} whose functional value is 1 and the remaining $(2m-3)$ edges have functional values 0.

$$\text{Therefore} \sum_{e \in N(l_{ij})} f(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-3)times} + 1 = 1.$$

Suppose $f(l_{ij}) = 1$, $j = 1$ and $m > 3$. Then $N(l_{i1})$ contains l_{i2} whose functional value is 1 and an edge h_{i1} whose functional value is 1 and the remaining $(2m-4)$ edges have functional values 0.

$$\text{Therefore} \sum_{e \in N(l_{ij})} f(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-4)times} + 1 + 1 = 2.$$

Suppose $f(l_{i1}) = 1, j \neq 1$ and $m = 3$.

Suppose $j = 2$.

Then $N(l_{i2})$ contains an edge l_{i1} whose functional value is 1 and an edge l_{i3} whose functional value is 0 and the remaining edges have functional values 0.

Similarly, if $j = 3$, then $N(l_{i3})$ contains l_{i1} and h_{i1} whose functional values are 1 and an edge l_{i2} whose functional value is 0 and the remaining edges have functional values 0

Therefore $$\sum_{e \in N(l_{ij})} f(e) = \underbrace{[0+0+\cdots+0]}_{(2m-3) times} + 1 = 1,$$

or

$$\sum_{e \in N(l_{ij})} f(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4) times} + 1 + 1 = 2.$$

Suppose $f(l_{ij}) = 1, j \neq 1$ and $m > 3$.

Then $N(l_{ij})$ contains one edge l_{ij} or two edges l_{ij} from T whose functional value is 1 and the remaining edges have functional values 0 and two edges h_{ij} whose functional values are 0. Then

$$\sum_{e \in N(l_{ij})} f(e) = \underbrace{[0+0+\cdots+0]}_{(2m-3) times} + 1 = 1$$

or

$$\sum_{e \in N(l_{ij})} f(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4) times} + 1 + 1 = 2.$$

Sub case (ii): Suppose $f(l_{ij}) = 0, j \neq 1, 2, \ldots, (m-2)$. That is $l_{ij} \notin T$. Then $N(l_{ij})$ may contain four edges l_{ij} or three edges l_{ij} or two edges l_{ij} or one edge l_{ij} from T whose functional value is 1 and may or may not contain an edge h_{i1}.

For all possibilities we get

$$\sum_{e \in N(l_{ij})} f(e) = 1 + \underbrace{[0+0+\cdots+0]}_{(2m-3) times} = 1, \text{ if } N(l_{ij}) \text{ does not contain } h_{i1},$$

or

$$\sum_{e \in N(l_{ij})} f(e) = 1 + \underbrace{[0+0+\cdots+0]}_{(2m-4) times} + 1 = 2, \text{ if } N(l_{ij}) \text{ contains } h_{i1}.$$

and

$$\sum_{e \in N(l_{ij})} f(e) = 1 + 1 + \underbrace{[0+0+\cdots+0]}_{(2m-4) times} = 2, \text{ if } N(l_{ij}) \text{ does not contain } h_{i1},$$

or

$$\sum_{e \in N(l_{ij})} f(e) = 1 + 1 + \underbrace{[0+0+\cdots+0]}_{(2m-5) times} + 1 = 3, \text{ if } N(l_{ij}) \text{ contains } h_{i1}.$$

and

$$\sum_{e \in N(l_{ij})} f(e) = 1 + 1 + 1 + \underbrace{[0+0+\cdots+0]}_{(2m-5) times} = 3, \text{ if } N(l_{ij}) \text{ does not contain } h_{i1},$$

or

$$\sum_{e \in N(l_{ij})} f(e) = 1 + 1 + 1 + \underbrace{[0+0+\cdots+0]}_{(2m-6) times} + 1 = 4, \text{ if } N(l_{ij}) \text{ contains } h_{i1},$$

and

$$\sum_{e \in N(l_{ij})} f(e) = 1 + 1 + 1 + 1 + \underbrace{[0+0+\cdots+0]}_{(2m-6) times} = 4, \text{ if } N(l_{ij}) \text{ does not contain } h_{i1}.$$

or

$$\sum_{e \in N(l_{ij})} f(e) = 1 + 1 + 1 + 1 + \underbrace{[0+0+\cdots+0]}_{(2m-7) times} + 1 = 5, \text{ if } N(l_{ij}) \text{ contains } h_{i1}.$$

Case 3: Let $h_{i1} \in C_n \odot K_m$ be such that $adj(h_{i1}) = 2m$.

Then $f(h_{i1}) = 1$ and $N(h_{i1})$ contains two edges of C_n, $(m-1)$ edges h_{ij} of G and

$(m-1)$ edges l_{ij} in the i^{th} copy of K_m. Among these $(m-1)$ edges of K_m, there is an edge l_{ij} from T such that $f(l_{i1}) = 1$. The remaining edges have functional value 0.

So $$\sum_{e \in N(h_{i1})} f(e) = 0 + 0 + \underbrace{[0+0+\cdots+0]}_{(2m-3)-times} + 1 = 1.$$

Sub Case (iii): Let $h_{ij} \in C_n \odot K_m, j \neq 1$.

Here $adj(h_{ij}) = 2m$.

Then $N(h_{i1})$ contains two edges of C_n, $(m-1)$ edges h_{ij} of G and one edge l_{ij} or two edges l_{ij} or no edge l_{ij} from T.

For all these possibilities we get

$$\sum_{e \in N(h_{ij})} f(e) = 0 + 0 + \underbrace{[0+0+\cdots+0]}_{(2m-4) times} + 1 + 1 = 2,$$

Or

$$\sum_{e \in N(h_{ij})} f(e) = 0 + 0 + \underbrace{[0+0+\cdots+0]}_{(2m-5) times} + 1 + 1 + 1 = 3,$$

or

$$\sum_{e \in N(h_{ij})} f(e) = 0 + 0 + \underbrace{[0+0+\cdots+0]}_{(2m-3)-times} + 1 = 1.$$

Therefore it follows that for all possibilities of edges $e \in G$, we have

$$\sum_{e \in E(G)} f(e) \geq 1.$$

Hence f is a total edge dominating function of G.

We now check for the minimality of f.

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Define a function $g: E \to [0,1]$ by

$$g(e) = \begin{cases} r, \text{for } h_{i1} \in T, \\ 1, \text{ for } e \in T - \{h_{i1}\}, \\ \text{o, otherwise,} \end{cases}$$

where $0 < r < 1$.

Clearly $g < f$.

Now we check for the minimality of f.

Case (i): Let $e_i \in C_n, i = 1,2, \ldots, n$ be such that $adj(e_i) = 2m + 2$.

Sub case 1: Let $h_{i1} \in N(e_i)$. Then

$$\sum_{e \in N(e_i)} g(e) = 0 + 0 + \underbrace{[0 + 0 + \cdots + 0]}_{(2m-2) times} + r + 1 = r + 1.$$

Sub case 2: Let $h_{i1} \notin N(e_i)$. Then

$$\sum_{e \in N(e_i)} g(e) = 0 + 0 + \underbrace{[0 + 0 + \cdots + 0]}_{(2m-2) times} + 1 + 1 = 2.$$

Case (ii): Let $l_{ij} \in i^{th}$ copy of K_m be such that

$$adj(l_{ij}) = 2m - 2.$$

Sub case (iv): Suppose $f(l_{ij}) = 1, j = 1$ and $m = 3$.

Sub case 1: Let $h_{i1} \in N(l_{ij})$. Then

$$\sum_{e \in N(l_{ij})} g(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-4) times} + r + 0 = r < 1.$$

Suppose $f(l_{ij}) = 1$, $j = 1$ and $m > 3$. Then

$$\sum_{e \in N(l_{ij})} g(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-4) times} + r + 1 = r + 1.$$

Sub case 2: Let $h_{i1} \notin N(l_{ij})$. Then

$$\sum_{e \in N(l_{ij})} g(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-4) times} + 1 + 1 = 2.$$

Suppose $f(l_{ij}) = 1, j \neq 1$ and $m > 3$.

If $h_{i1} \in N(l_{ij})$, then

$$\sum_{e \in N(l_{ij})} g(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-3) times} + r = r < 1,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = [\underbrace{0 + 0 + \cdots + 0}_{(2m-4) times}] + r + 1 = r + 1.$$

If $h_{i1} \notin N(l_{ij})$, then

$$\sum_{e \in N(l_{ij})} g(e) = \underbrace{[0 + 0 + \cdots + 0]}_{(2m-3) times} + 1 = 1,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = [\underbrace{0 + 0 + \cdots + 0}_{(2m-4) times}] + 1 + 1 = 2.$$

Sub case (v): Suppose $f(l_{ij}) = 0, j \neq 1$. Then $j = 2,3, \ldots, m - 2$.

Sub case 1: Let $h_{i1} \in N(l_{ij})$. Then

$$\sum_{e \in N(l_{ij})} g(e) = r + \underbrace{[0 + 0 + \cdots + 0]}_{(2m-3) times} = r < 1,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = r + 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-4) times}] = r + 1,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = r + 1 + 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-5) times}] = r + 2,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = r + 1 + 1 + 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-6) times}] = r + 3.$$

Sub case 2: Let $h_{i1} \notin N(l_{ij})$. Then

$$\sum_{e \in N(l_{ij})} g(e) = 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-3) times}] = 1,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = 1 + 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-4) times}] = 2,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = 1 + 1 + 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-5) times}] = 3,$$

or

$$\sum_{e \in N(l_{ij})} g(e) = 1 + 1 + 1 + 1 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-6) times}] = 4.$$

Case (iii): Let $h_{ij} \in C_n \odot K_m$, $adj(h_{ij}) = 2m$, $j \neq 1$.

Sub case 1: Let $h_{i1} \in N(h_{ij})$. Then

$$\sum_{e \in N(h_{ij})} g(e) = 0 + 0 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-4) times}] + r + 1 = r + 1,$$

or

$$\sum_{e \in N(h_{ij})} g(e) = 0 + 0 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-5) times}] + r + 1 + 1 = r + 2,$$

or

$$\sum_{e \in N(h_{ij})} g(e) = 0 + 0 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-3) times}] + r = r < 1.$$

Sub case 2: Let $h_{i1} \notin N(h_{ij})$. Then

$$\sum_{e \in N(h_{ij})} g(e) = 0 + 0 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-4) times}] + 1 + 1 = 2,$$

or

$$\sum_{e \in N(h_{ij})} g(e) = 0 + 0 + [\underbrace{0 + 0 + \cdots + 0}_{(2m-5) times}] + 1 + 1 + 1 = 3,$$

or

$$\sum_{e\epsilon N(h_{ij})} g(e) = 0 + 0 + \underbrace{[0 + 0 + \cdots + 0]}_{(2m-3)times} + 1 = 1.$$

We see that $\sum_{e\in E(G)} g(e) < 1$ for some $e \in E(G)$.

So g is not a total edge dominating function.

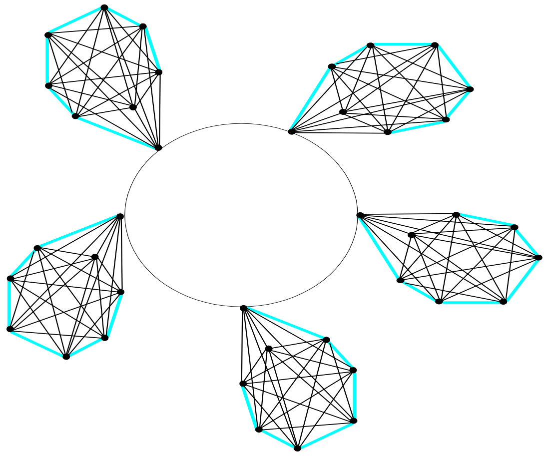
Since $g$is defined arbitrarily, it follows that there exists no $g < f$such that g is a total edge dominating function.

Hence f is a minimal total edge dominating function of $G = C_n \odot K_m$.

3. ILLUSTRATIONS

3.1 Minimal Total Edge Dominating Set

Theorem 2.2

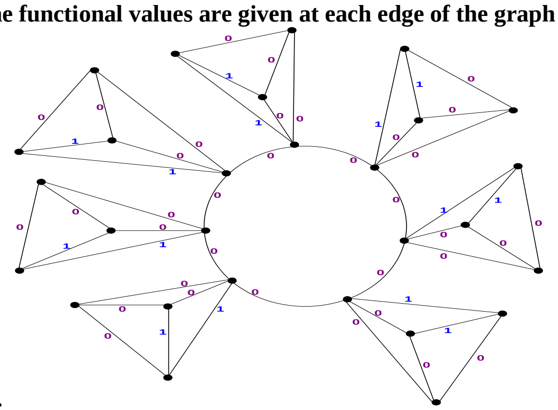


$G = C_5 \odot K_7$

3.2 Minimal Total Edge Dominating Function

Theorem 2.3

The functional values are given at each edge of the graph G.



$G = C_7 \odot K_3$

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