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An Alternative Algorithm for Solving Pure Integer Linear Programming Problems Having Two Variables

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ABSTRACT
An alternative algorithm is proposed, based on parametrization for solving a special class of integer linear programming (ILP) problems when the objective function is linear and the constraints are in the form of linear inequality. Although there are popular methods in the literature having widespread impact they are known to have some difficulties in terms of computation. To overcome these difficulties, a parameter-based algorithm that could be applied reliably and easily to (ILP) problems with two variables and no restriction on the constraints is proposed. The flow of the algorithm provides a set constructed by variable values that depend on the parameter. Thus, the solution satisfying the constraints can be selected easily from this set. The proposed algorithm is remarkable in that it can be applied easily even when the number of restrictions increases.

General Terms
Computational Mathematics, Optimization

Keywords
Linear integer programming, Linear Diophantine equations, optimal hyperplane

1. INTRODUCTION
Linear programming (LP) has been emerged based on management sciences to formulate and solve optimization problems. Linear programming is a tool for solving optimization problems. Solving linear programming problems efficiently is impressive for the scientists. Many researchers have proposed different algorithms to solve linear programming problems. Generally, the decision variables in LP are considered to be fractional, as continuous. However, in many real life applications, such as capital budgeting, production planning, capacity planning, scheduling and chemical engineering process; to use the fractional values of the decision variables will not be meaningful and realistic. Integer programming models that is originated from the applications of mathematical programming to support managerial decisions play an important role in operations research. Thus, in practice, a large variety of different real life problems can be formulated as integer optimization problems.

Rounding off the solution without violating any of the constraints can results a value that is considerable far from the original optimum value, for the objective function. Integer linear programming (ILP) enables to solve the constrained optimization problems with integer variables, where the objective function and the constraints are linear. As a type of mathematical programming, ILP is a linear programming problem in which at least one of the variables is restricted to integer values whereas pure Linear Integer programming problem whose variables are all restricted to be integer emphasizes an IP.

The main purpose of [2] demonstrates the feasibility of the approach for the general integer programming problem. It investigates the role of extreme points in the solution of the general ILP.

In [3], a variable reduction method for a classes of pure integer linear programming problems, which is better than the cutting plane method and the branch bound method, is introduced based on simple mathematical concepts. An algorithm to locate all alternative optimal solutions of the ILP problem is developed in [4] because of multiple solutions enable the flexibility for making a decision.[5] suggests by formulating a integer linear programming problem to solve a general daily staff scheduling problem utilizing the 24-hour work shifts .The paper [6] presents a survey of methods and approaches for solving integer linear problems which are belong to the class of NP-hard optimization problems, during the last 50 years. For ILP problems, in [7], only weak duality is guaranteed. Even if an optimal feasible solution cannot be found, they can have a good bound for every ILP. And also, they implemented the algorithm for solving Capital budgeting and scheduling type problems. An effective method is presented in [8] for solving ILP problems, by checking whether a feasible point is an optimal solution of the ILP. A new algorithm for solving integer programming (ILP) problems that is based on algebraic geometry is proposed in [10].They extend the approach to the feasibility and optimization problems of general IPs. The results can be presented as a generalization of Farkas' lemma to integer programming. In [11], a new iterative method based on the conjugate gradient projection method using of the spirit of Gomory cut for solving the ILP problems when the objective function and the set of constraints are in the form of linear inequality constraints is presented. A fixed point iterative method for integer programming is developed in [12] that can be applied to compute all integer or mixed-integer points in a polytope and directly extended to convex nonlinear integer

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programming. In [13], an evolutionary algorithm for the solution of pure integer linear programs having all the variables are fixed by the evolutionary system is introduced. The algorithm proposed does not require the solution of continuous linear programs.

In this paper, a pure integer linear programming model with only two variable and no restriction on the numbers of the constraints, is presented whose solution can be directly obtained using the standard simplex method. An appropriate iterative computational approach in order to overcome these problems is proposed based on simple mathematical concept. In this context, the parameterization algorithm to eliminate the computational difficulty is served as an effective tool.

This paper is organized as follows: Section 2 presents required information. In Section 3, the proposed approach is handled. Section 4 and Section 5 consist of the numerical example and conclusions, respectively.

2. PRELİMİNARİES

In this section, brief required information are presented.

Definition 1[9]: The mathematical formulation of an ILP problem is described below:

$$P_1:\begin{cases} \text{Max(Min)}\sum_{j=1}^{n} c_j x_j \\ \sum_{i=1}^{m}\sum_{j=1}^{n} a_{ij}x_j \le b_i \\ x_j \ge 0 \text{ and ınteger; where } (i=1,2,...,m;\ j=1,2,...,n) \end{cases}$$

Definition 2:Consider the objective hyperplane

$$\sum c_j x_j = z$$

Where each $c_j \in Z$ which is a linear Diophantine equation in integers [1].

$$d = gdc(c_j, c_j \ne 0;\ j = 1,2,...,n).$$

It has an integer solution if and only if $d|z$. Also, if a linear Diophantine equation has an integer solution; then there will be infinitely many solution for this equation [1].

Theorem 1: $(x_1, x_2, ..., x_n, z)$ is a solution of the problem if and only if $(x_1, x_2, ..., x_n)$ satisfies all constraints of P_1.

Theorem 2: Let z be an integer. Let S denote the set of all feasible solutions to the ILP. If $S \cap \{x / cx = z\}$ is non empty, then the optimum solution to the ILP will lie on the hyperplane $cx = z$ [1].

3. A PROPOSED ALGORİTHM FOR SOLVİNG THE INTEGER LİNEAR PROGRAMMING PROBLEMS HAVİNG TWO VARİABLES

The solution algorithm for finding the solution of an ILP problem having two variables is presented below:

Step 0: Load the LP problem P_1.

Step 1: Solve the relaxed LP problem P_1 to find the optimal solution.

Step 2: If the optimal point (x_1, x_2) is integer, it is the desired solution. Stop. Otherwise, go to step 3.

Step 3: Assign the integer part of the optimal value Z of P1 to the expression $\sum_{j=1}^{2} c_j x_j$.

Step 4: In $\sum_{j=1}^{2} c_j x_j = Z$, specify any of the variables as a parameter and determine the other variable as parametrically.

Step 5: Reconstruct the constraints parametrically.

Step 6: Determine the interval of parameter, considering the reconstructed constraints.

Step 7: If it is meaningful, specify each pair of variables (x1, x2) for each parameter value that is from the interval and go to step 8. Otherwise, decrease (increase) the optimal value Z one unit and considering new optimal value go to step 3.

Step 8: Choose integer variable pair(s) (x1, x2).

Step 9: Check whether the resulting pair(s) (x1, x2) satisfy the constraints. If yes, check whether the determined (x1, x2) is the only solution that satisfies the constraints. If yes, it is the optimal solution.

Stop. Otherwise, choose one of them who is maximizing (minimizing) the objective function. Stop. If there are solutions satisfying the constraints more than one, they will be alternative solutions. If no one provides the constraints, decrease (increase) the optimal value Z one unit and considering new optimal value, go to step 3.

An algorithm having easy implementation to solve integer linear programming model having two variables is developed. All these steps have been aimed of optimizing readily the problems meeting numerous constraints with two variables.

4. NUMERICAL EXPERIMENT

Example 4.1.

Solve the following LP problem.

Step 0:

P_1 Max $x_1 + 5x_2$

Subject to

$x_1 + 10x_2 \le 20$

$x_1 \le 2$

$x_1 \ge 0,\ \ x_2 \ge 0$ and integers

Step 1: By solving the relaxed LP problem $P_1;\ (x_1, x_2) = (2.1, 1.8)$ and optimal value $Z = 11$ are obtained.

Step 2: There is no integer solution, go to step 3.

Step 3: Z=11 assigned to $x_1 + 5x_2$.

Step 4: By assigning $x_1 = t$, in $x_1 + 5x_2 = 11$, $x_2 = \frac{11-t}{5}$ is obtained.

Step 5-6: Considering the parametric variables, the constraints is reconstruct as follows:

$t \geq 2;\ t \leq 2$

Step 7: Reconstructed constraints present t as 2

Step 8:And so, the solution is determined as $(x_1, x_2) = (2, 9/5)$

Since it is not an integer solution, the optimal value Z is reduced one unit; i.e. Z is taken as 10; and, go to step 3.

Thus, $0 \leq t \leq 2$ is determined from $x_1 + 5x_2 = 10$. When the parameter values of $0 \leq t \leq 2$ are examined, it is clear that $(0,2)$ produces an integer solution which gives the optimal value as 10.

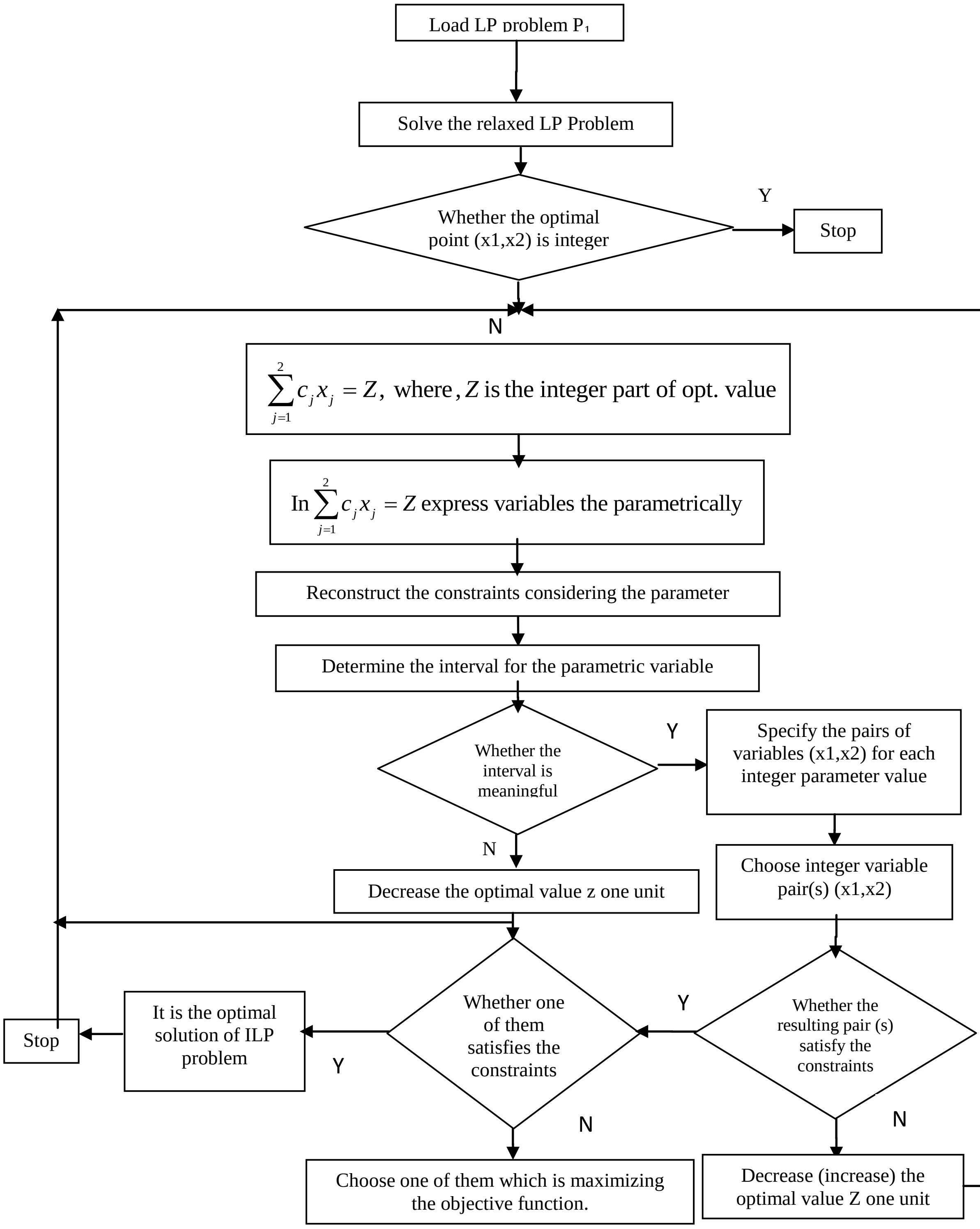

Fig.1 The algorithm for integer linear programming model having two variables

5. CONCLUSION

In this study, by using basic algebraic geometrical knowledge and determining suitable parametrization; traditional simplex method has been activated to solve the ILP problems with only two variable and no restriction on the numbers of the constraints. In terms of understanding and implementing easily, it is clear that the presented algorithm has a notable advantage to provide an optimal solution as an algebraic computational approach. Due to the computational complexity, many real problems having two variables and large constraints cannot be solved using existing approaches. The process which can be used independently of the size of the constraints is presented with a numerical example to demonstrate the reliability and efficiency of the process. The solution approach can be developed to solve modeled real life problems that involves three or more variables. Also, the presented optimization algorithm can be implemented to a software package easily.

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