International Journal of Applied Information Systems (IJAIS) – ISSN : 2249-0868
Foundation of Computer Science FCS, New York, USA
Volume 12– No. 29, April 2020 – www.ijais.org

Co-regular Total Domination in Graphs

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ABSTRACT
A total dominating set D of a graph $G=(V,E)$ is a coregular total dominating set if the induced subgraph $<V-D>$ is regular. The coregular total domination number $\gamma_{crt}(G)$ of G is the minimum cardinality of a coregular total dominating set. In this paper, we study its exact values for some standard graphs and many bounds on $\gamma_{crt}(G)$ were obtained. Its relation with other different domination parameter investigated.

Keywords
Graph, Domination number, Coregular total domination number

Subject classified number: AMS-05C69, 05C70.

1. INTRODUCTION
Let $G=(V,E)$ be a finite, undirected, simple graph with p vertices and q edges. For any undefined term or notation in this paper can be found in Harary [4]. A vertex in a graph dominates itself and its neighbors. For G, let Δ and δ be the maximum and minimum degree. In general we use $<X>$ to denote the subgraph induced by the set of vertices X and $N(v)$ and $N[v]$ denote the open and closed neighborhood of a vertex, respectively. The notation $\alpha_\circ(G)\alpha_1(G)$ is the minimum cardinality of vertices(edges) in a vertex(edge) cover of G. Also $\beta_\circ(G)\beta_1(G)$ is the minimum cardinality of vertices(edges) in a maximal independent set of a vertex (edge) of G. The degree of an edge $e=uv$ of G is defined by $\deg(e)=\deg(u)+\deg(v)-2$ and $\delta'(G)\Delta'(G)$ is the minimum (maximum) degree among the edges of G(the degree of an edge is the number of edges adjacent to it). $\chi(G)\chi(G')$ is the minimum for which G has an $n-$vertices($n-$edges) colourings. We begin with some standard definitions from domination theory.

A set $S\subseteq V$ is a dominating set of G if every vertex not in S is adjacent to a vertex in S. The domination number of G is denoted by $\gamma(G)$ is the minimum cardinality of a dominating set. For detail on $\gamma(G)$ studied in [6,7].

A dominating set $S\subseteq V$ of G is connected dominating set if the induced subgraph $<S>$ has one component .The connected domination number $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set of G.

A dominating set $S\subseteq V(G)$ is a restrained dominating set of G if every vertex not in S is adjacent to a vertex in $V-S$. The restrained domination number of G is denoted by $\gamma_r(G)$ is the smallest cardinality of a restrained dominating set of G. The concept of restrained domination in graphs introduced by Domke et.al (1999)see[3].

Analogously, a dominating set D of a graph G is a cototal dominating set if the induced subgraph $<V-D>$ has no isolated vertices. The cototal domination number number $\gamma_{ct}(G)$ is the minimum cardinality of a cototal dominating set of G.

A dominating set $S\subseteq V$ of G is split dominating set if the induced subgraph $<V-S>$ has more than one component. The split domination number $\gamma_s(G)$ of G is the minimum cardinality of a split dominating set. For details see[8].

In this paper, we introduce the new concept in domination theory. A total dominating set D of G is a coregular total dominating set if the induced subgraph $<V-D>$ is regular. The coregular total domination number $\gamma_{crt}(G)$ of G is the minimal cardinality of a coregular total dominating set of G.

2. RESULTS
We need the following theorems

Theorem A [8]: For any connected graph G, $\left\lceil\frac{p}{\Delta(G)+1}\right\rceil\leq\gamma(G)$.

Theorem B [1]: Let G be a connected graph of order p, then $\gamma'(G)\leq\left\lceil\frac{p}{2}\right\rceil$.

Theorem C [8]: For any graph G with end vertex $\gamma(G)=\gamma_s$(G).

The following theorem relates $\gamma_{crt}(G)$ interms of $\gamma_g(G)$ and edges of G.

Theorem 1: For any connected (p,q) graph G with $p\geq 3$ vertices,

$$2\gamma_g(G)-\gamma_{crt}(G)\leq q.$$

Proof: Let $E=\{e_1,e_2,\dots\dots\dots\dots,e_n\}\subseteq E(G)$ be the edge set of G. Let $\{v_1,v_2,\dots\dots\dots,v_p\}$ be the vertex set of G and let $A=\{v_1,v_2,\dots\dots\dots\dots,v_p\}$ be the set of all nonendvertices which are adjacent to endvertices in G. Suppose $D=\{v_1,v_2,\dots\dots\dots\dots,v_m\}$,$m\leq p$ be a dominating set of G. Such that $N[D]=V(G)$ and $N[D]=N(\bar{G})$. Then D is a dominating set for both G and $\bar{G}$. Suppose the induced subgraph $<\{D\}\cup\{A\}>$ do not have isolated vertex then $\{D\}\cup\{A\}$ is a $\gamma_t(G)$ set. Further there exists $D'=[V(G)-\{D\}\cup\{A\}]$ and $<D'>$ is regular then $\{D'\}$ is $\gamma_{crt}(G)$ set of G. Hence $2|D|-|D'|\leq|E|$ which gives , $2\gamma_g(G)-\gamma_{crt}(G)\leq q$.

Theorem 2: For any connected (p,q) graph G with $p\geq 3$ vertices,

$$\gamma_{crt}(G)+diam(G)\leq p+\alpha_\circ(G).$$

Proof: Let $B=\{v_1,v_2,\dots\dots\dots\dots,v_m\}\subseteq V(G)$,$deg(v_i)\geq 2,\forall\, v_i\epsilon B$ be the set of vertices which contains $B'=$

$\{v_1, v_2, \ldots \ldots \ldots, v_k\}$ such that the set of vertices B' covers all the edges in G.There exists an edge set $E_1 \subseteq E(G)$ constitute the longest path between two distinct vertices $u, v \in V(G)$ such that $dist(u,v) = diam(G)$. Suppose $D = \{u_1, u_2, \ldots \ldots \ldots ., u_m\} \subseteq V(G)$ be the minimal set of vertices which covers all the vertices in G and the induced subgraph $< D >$ has no isolated vertex then D itself is a $\gamma_t(G)$ set of G. Suppose the subgraph $D' = [V(G) - D]$ and $< D' >$ is regular then D' forms a $\gamma_{crt}(G)$ set of G. Therefore it follows that $|D'| + diam\,(G) \leq |V(G)| + |B'|$ and hence $\gamma_{crt}(G) + diam\,(G) \leq p + \alpha_\circ(G)$.

The following theorem relates the domination number of G and $\gamma_{crt}(G)$.

Theorem 3: For any connected (p,q) graph G $\gamma_{crt}(G) \geq \gamma(G)$.

Proof: It is easy to see that $\gamma_t(G) \geq \gamma(G)$ and for $\gamma_{crt}(G)$ the above result may also exists for any connected graph.

Theorem 4: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\gamma_{crt}(G) \geq \left\lceil \frac{p}{\Delta(G)+1} \right\rceil.$$

Proof: By Theorem A and also by Theorem 3 we have the required result.

The concept of Roman domination function (RDF) was introduced by E. J. Cockayne, P.A. Dreyer, S. M. Hedetniemi and S. T. Hedetniemi in [2]. A Roman dominating function on a graph $G = (V,E)$ is a function $f: V \to \{0,1,2\}$ satisfying the condition that every vertex u for which $f(u) = 0$ is adjacent to at least one vertex of v of G for which $f(v) = 2$. The weight of a roman dominating function is the value $f(v) = \sum_{v \in V} f(x)$. The Roman domination number of a graph G, denoted by $\gamma_R(G)$ equals the minimum weight of a Roman dominating function on G.

Theorem 5: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\gamma_t(G) + \gamma_R(G) - 3 \leq 2\gamma_{crt}(G) + \chi(G).$$

Proof : Let $F = \{v_1, v_2, \ldots \ldots \ldots \ldots ., v_k\} \subseteq V(G)$ be the set of all endvertices in G and $V' = V - F$. Suppose $D' \subseteq V'$ be a minimal dominating set of G. Further if for some $\{v_i\} \in N(D')$ and $< D' \cup N(v_i) >$ has no isolates, then $D' \cup N\{v_i\}$ forms a minimal total dominating set of G. If $\{v_i\} = \emptyset$, then there exists at least one vertex $v \in F$ such that $D' \cup N\{v\}$ forms a total dominating set of G. Suppose $f: V \to \{0,1,2\}$ and partitation the vertex set $V(G)$ in to (V_0, V_1, V_2) by f with $|V_i| = n_i$ for $i = 0,1,2,$.Suppose the set V_2 dominates V_0 then $H = V_1 \cup V_2$ forms a minimal roman dominating set of G. Further if $< V - D' \cup \{v_i\} >$ is regular then $D' \cup \{v_i\}$ is coregular total dominating set . Harary [4] has proved the chromatic number $\chi(G) \leq 1 + \Delta(G)$ and by Theorem 4, we have $|D' \cup N\{v\}| + |H| - 3 \leq 2|D' \cup \{v_i\}| + \chi(G)$ which gives, $\gamma_t(G) + \gamma_R(G) - 3 \leq 2\gamma_{crt}(G) + \chi(G)$.

A restrained dominating set D of a graph $G = (V,E)$ is a coregular restrained dominating set if the induced subgraph $< V - D >$is regular. The coregular restrained domination number $\gamma_{crr}(G)$ of G is the minimum cardinality of a coregular restrained dominating set see[10].

The following Theorem relates with split domination number, coregular restrained domination number $\gamma_{crt}(G)$ and Roman domination number.

Theorem 6: For any connected (p,q) graph G with $p \geq 3$ vertices, $\gamma_{crt}(G) - \gamma_s(G) \leq \gamma_{crr}(G) + \gamma_R(G) - 1$ and $G \neq K_p$, $G \neq K_{1,p}$.

Proof : Let D be a minimal dominating set of G . Suppose $< V - D >$ is disconnected then D itself is a split dominating set of G. Further, if G has a set $B = \{v_1, v_2, \ldots \ldots \ldots \ldots ., v_p\}$ a set of end vertices in G . Then for $\forall\, v_i \in [V(G) - D \cup B]$ is adjacent to at least one vertex of $D \cup B$ and at least one vertex of $V(G) - \{D \cup B\}$. So $\{D \cup B\}$ is a minimal restrained dominating set of G. Suppose $< V - \{D \cup B\} >$ is regular, then $\{D \cup B\}$ is a $\gamma_{crr}(G)$ set of G. For γ_R, let $f = (V_0, V_1, V_2)$ be any γ_R- function of G.Then $V_1 \cup V_2$ is a $\gamma_R(G)$ set of G. Let $D_1 = \{v_1, v_2, \ldots \ldots \ldots \ldots ., v_k\} \subseteq V(G)$ be the set of all nonendvertices in G. Suppose $D_2 \subseteq D_1$ be the minimum set of vertices in G and if $\deg(v_j) \geq 1$, $\forall\, v_j \in D_2$. Then D_2 forms a total dominating set of G. Otherwise if $\deg(v_j) < 1$ attach the vertices $w_i \in N(v_j)$ to make $\deg(v_j) \geq 1$, such that $< D_2 \cup \{w_i\} >$ has no isolates. Clearly $< D_2 \cup \{w_i\} >$ forms a minimal total dominating set of G. If the induced subgraph $< V(G) - D_2 \cup \{w_i\} >$ is regular, then $D_2 \cup \{w_i\}$ is a coregular total dominating set of G. Hence $|D_2 \cup \{w_i\}| - |D| \leq |D \cup B| + |V_1 \cup V_2|$-1 which gives ,$\gamma_{crt}(G) - \gamma_s(G) \leq \gamma_{crr}(G) + \gamma_R(G) - 1$.

Theorem 7: A coregular total dominating set $D' \subseteq V(G)$ is minimal if and only if for each vertex $x \in D'$ one of the following condition holds

a) There exists a vertex $y \in V(G) - D'$ such that $N(y) \cap D' = x$
b) x is not an isolated vertex in $< D' >$
c) $< V(G) - D' >$ is regular.

Proof : Suppose D' is a minimal coregular total dominating set of G and there exists a vertex $x \in D'$ such that x does not hold only of the above conditions. Then for some vertex w, the set $D_1 = D' - \{w\}$ forms a coregular total dominating set in G by condition (a) and (b). Also by (c) $< V(G) - D' >$ is regular.This implies that D' is coregular total dominating set of G, a contradiction.

Conversly, suppose $\forall\, x \in D'$, there exists a vertex $y \in V(G) - D'$ and condition (a) holds. Then $N(y) \cap D' = x$. For condition (b), $\deg(x) \geq 1, \forall\, x \in D'$. Further if the condition (c) holds and $< V(G) - \{D' - x\} >$is not regular. Clearly D' is a coregular tota dominating set of G.

Theorem 8: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\gamma_{crt}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 2.$$

Proof: Let $D_1 = \{v_1, v_2, \ldots \ldots \ldots \ldots ., v_n\}$ is a dominating set of G. Suppose $V_1 = V(G) - D_1$ .Further $D_2 \subset V_1$ and $< D_1 \cup D_2 >$ has no isolates and $N[D_1 \cup D_2] = V(G)$ then $D_1 \cup D_2$ forms a total dominating set of G. If $D_3 = [V(G) - D_1 \cup D_2]$ and $< D_3 >$ is regular then $\{D_3\}$ is $\gamma_{crt}(G)$ set of G. Also by Theorem B, $\gamma'(G) \leq \left\lceil \frac{p}{2} \right\rceil$. Clearly it follows that $|D_3| \leq \left\lceil \frac{p}{2} \right\rceil + 2$ and hence $\gamma_{crt}(G) \leq \left\lceil \frac{p}{2} \right\rceil + 2$.

Proposition 1: For any graph G with $p \geq 3$ vertices

$$\gamma_{crt}(K_p) = \gamma_{crt}(K_{1,p}) = \gamma_t(G).$$

Theorem 9: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$2\gamma_{crt}(G) - \beta_1(G) \leq \gamma_c(G) + \gamma_r(G) + \delta'(G).$$

Proof: Let $B = \{e_1, e_2, \ldots\ldots\ldots\ldots, e_n\} \subseteq E(G)$ be the minimal independent set with $|B| = \beta_1(G)$. Suppose $S = \{v_1, v_2, \ldots\ldots\ldots, v_k\} \subseteq V(G)$ be the minimal set of vertices which covers all the vertices in G. Clearly S forms a minimal dominating set of G. If the subgraph $< S >$ has one component then S itself is $\gamma_c(G)$ set. On the other hand $B_1 = \{u_1, u_2, \ldots\ldots\ldots\ldots, u_n\}$ be the set all endvertices in G such that $S_1 \subseteq S$ then $B_1 \cup S_1$ forms a minimal restrained dominating set of G. Let $e \epsilon E(G)$ with $\deg(e) = \delta'(G)$. Further for some $v_j \epsilon N(S)$ and the induced subgraph $< S \cup \{v_j\} >$ does not contains an isolates then $S \cup \{v_j\}$ is a total dominating set of G. If $D = [V(G) - S \cup \{v_j\}]$ and $< D >$ is regular then $\{D\}$ is a $\gamma_{crt}(G)$ set of G, it follows that $2|D| - |B| \leq |S| + |B_1 \cup S_1| + \delta'(G)$ which gives, $2\gamma_{crt}(G) - \beta_1(G) \leq \gamma_c(G) + \gamma_r(G) + \delta'(G)$.

Observation 1: Let T be a tree, then each $\gamma_{crt}(G)$ set is $\gamma_t(G)$ set.

Observation 2: Every support vertex of a graph is in every $\gamma_{crt}(G)$ set.

A split dominating set $D \subseteq V(G)$ is a coregular split dominating set if the induced subgraph $< V - D >$ is regular and disconnected. The minimum cardinality of such a set is called a coregular split domination number and is denoted by $\gamma_{crs}(G)$. For detail see[9].

We establish the relationship with coregular split domination number with Coregular total domination number of G.

Theorem 10: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\gamma_{crt}(G) \leq \gamma_{crs}(G) + \Delta(G) \text{ and } G \neq K_p.$$

Proof: Suppose $G = K_p$. Then by definition $\gamma_{crt}(G)$ set does not exists. Now consider D_1 be a dominating set of G and $V_1 = V(G) - D_1\ \forall\, v_i \epsilon V_1$ if $< D_1 \cup \{v_i\} >$ has no isolates. Then $< D_1 \cup \{v_i\} >$ forms a minimal total dominating set of G. Further if $V_2 = [V(G) - D_1 \cup \{v_i\}]$ and $\forall\, v_i \in < V_2 >$ has same degree then $\{V_2\}$ is a $\gamma_{crt}(G)$ set of G. On the other hand let $B = \{v_1, v_2, \ldots\ldots, v_m\}$be the set of all nonendvertices $\forall\, B' \epsilon D_1$ such that $< V(G) - B' >$ is disconnected and is regular then B' forms a $\gamma_{crt}(G)$ set of G. Suppose there exists at least one vertex v of maximum degree $\Delta(G)$ it follows that $|V_2| \leq |B'| + \Delta(G)$ which gives , $\gamma_{crt}(G) \leq \gamma_{crs}(G) + \Delta(G)$.

Theorem 11: For any connected (p,q) graph G with $p \geq 3$ vertices, $2\alpha_1(G) - \gamma_{crt}(G) \geq \gamma_s(G)$ and $G \neq K_p$. $G \neq W_p$ $(p = 4,6)$.

Proof: Suppose $= K_p$ $(p \geq 2)$. Then by the definition $\gamma_s(G)$ set does not exists. Further for the graph $G = W_p$,if $p = 4 and\ 6$ then ,$2\alpha_1(G) - \gamma_{crt}(G) = 2 < \gamma_s(G)$. Let $S = \{e_1, e_2, \ldots\ldots\ldots\ldots, e_n\}$be the set of all endedges in G, where $K \subseteq E(G) - S$ such that $S \cup K$ be the minimal set of edges which covers all the vertices of G hence $|S \cup K| = \alpha_1(G)$. Further $A = \{v_1, v_2, \ldots\ldots, v_m\} \subseteq V(G)$ be the set of all endvertices in G. Let $B = V(G) - A$ and consider a set $F \subseteq B$ such that $< V(G) - F >$ is disconnected and if $N[F] = V(G)$. Hence F is a $\gamma_s(G)$ set of G . Let $A_1 \subseteq A$ and $B_1 \subseteq B$, now $< V(G) - \{A_1 \cup B_1\} >$ has no isolates . Clearly $\{A_1 \cup B_1\}$ is a total dominating set of G. Suppose $< V(G) - \{A_1 \cup B_1\} >$ is regular then $\{A_1 \cup B_1\}$ itself is a $\gamma_{crt}(G)$ set of G. Otherwise there exists $\{v_i\} \epsilon [V(G) - \{A_1 \cup B_1\}]$, now to see that $< V(G) - A_1 \cup B_1 \cup \{v_i\} >$ is regular . Hence $[A_1 \cup B_1 \cup \{v_i\}]$ is a $\gamma_{crt}(G)$ of G. Now $2|S \cup K| - |[A_1 \cup B_1 \cup \{v_i\}]| \geq |F|$, gives the required result $2\alpha_1(G) - \gamma_{crt}(G) \geq \gamma_s(G)$.

Theorem 12: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\gamma_{crt}(G) \leq q - \Delta'(G) + 1.$$

Proof: Let $E = \{e_1, e_2, \ldots\ldots\ldots\ldots, e_n\}$ be the edge set in G. Let $F = \{v_1, v_2, \ldots\ldots\ldots\ldots, v_m\}$ be the minimum set of vertices which covers all the vertices in G. Suppose $\deg(v_i) \geq 1$, $\forall\, v_i \epsilon F$ then F forms a $\gamma_t(G)$ set of G. Otherwise if $eg(v_i) < 1$, then attach the vertices $\forall\, v_j \epsilon N(v_i)$ to make $\deg(v_i) \geq 1$ such that $< F \cup \{v_j\} >$ has no isolated vertex . Clearly $[F \cup \{v_j\}]$ forms a minimal total dominating set of G. Further $[V(G) - F \cup \{v_j\}] = S$ and $< S >$ is regular, then $\{S\}$ is a coregular total dominating set of G. Suppose e be an edge of maximum degree $\Delta'(G)$ in G. Then $= uv \epsilon \{S\}$. Hence $|S| \leq |E| - \Delta'(G) + 1$ which gives , $\gamma_{crt}(G) \leq q - \Delta'(G) + 1$.

A dominating set $D \subseteq V(G)$ is a double dominating set of G, if each vertex in V is dominated by at least two vertices in D. Or a subset $D^d$of G is a double dominating set if for every vertex $v \epsilon V(G)$, $|N(v) \cap D^d| \geq 2$, that is v is in D^d and has at least one neighbor in D^d or v is in $V(G) - D^d$ has at least two neighbours in D^d. The double domination number $\gamma_{dd}(G)$ of G is the minimu cardinality of a double dominating set of G see[5].

Theorem 13: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\gamma_{crt}(G) + \gamma_{ct}(G) \leq 2\gamma_{dd}(G) + 1.$$

Proof: Let $A = \{u_1, u_2, \ldots\ldots\ldots, u_n\} \subseteq V(G)$ be the set of vertices. Suppose there exists a minimal set $B = \{u_1, u_2, \ldots\ldots\ldots\ldots, u_k\} \epsilon N(A)$ such that the subgraph $< A \cup B >$ has no isolated vertex. Further if $A \cup B$ covers all vertices in G, then $A \cup B$ forms a mnimal total dominating set of G. If $< V(G) - \{A \cup B\} >$ is regular then $\{A \cup B\}$ itself is a $\gamma_{crt}(G)$ set of G. Suppose $= \{v_1, v_2 \ldots\ldots, v_p\} \subseteq V(G)$, such that $N[D] = V(G)$. Then D is a dominating set of G and if $< V - D >$ has no isolates. Then D itself is a $\gamma_{ct}(G)$ set. Now consider $V_2 = V(G) - D$ and $D_2 = \{v_1, v_2, \ldots\ldots, v_j\} \subseteq V_2$,then $D^d = D \cup V_2$ forms a double dominating set of G. Since $|A \cup B| + |D| \leq 2|D \cup V_2| + 1$ which gives, $\gamma_{crt}(G) + \gamma_{ct}(G) \leq 2\gamma_{dd}(G) + 1$.

Theorem 14: For any connected (p,q) graph G with $p \geq 3$ vertices,

$$\left\lceil \frac{diam(G)+1}{2} \right\rceil \leq \gamma_{crt}(G).$$

Proof: Let $F = \{e_1, e_2, \ldots\ldots\ldots\ldots, e_k\} \subseteq E(G)$ be the minimal set of edges which constitute the longest path between any two distinct vertices $u, v \epsilon V(G)$ such that $dist(u,v) = diam(G)$. Further let D be the minimal dominating set in G. Suppose $V_1 = V(G) - D$ and $B \subseteq V_1$ such that $B \epsilon N(D)$ in . If $< V - D \cup B >$ is regular . Clearly $D \cup B$ is a $\gamma_{crt}(G)$ set of G. It follows that $\left\lceil\frac{diam(G)+1}{2}\right\rceil \leq |D \cup B|$ which gives, $\left\lceil\frac{diam(G)+1}{2}\right\rceil \leq \gamma_{crt}(G)$.

Theorem 15: For any graph G $\gamma_s(G) \leq \gamma_{crt}(G)$.

Proof: From Theorem 3, we have $\gamma(G) \leq \gamma_{crt}(G)$ … …._(1)

Also from Theorem C, $\gamma(G) = \gamma_s(G)$ ……………….. (2)

From (1) and (2) we have $\gamma_s(G) \leq \gamma_{crt}(G)$.

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