An Availability Measurement of Multi Component Power Plant by using Boolean Function Technique under different Distributions

Kashiful Haq
Associate Professor
Department of Computer Science
Meerut, India

Yusuf Perwej
Associate Professor
Department of Computer Science & Engineering
Delhi, India

ABSTRACT
The most essential part of our daily life is electricity. With the absence of it, every living human being will be sent to dark ages and in fact life itself would be incomplete and miserable without it. A power plant is an industrial facility that generates electricity from primary energy. Most power plants use one or more generators that convert mechanical energy into electrical energy in order to supply power to the electrical grid for society's electrical needs. In this research paper we have tried hard to calculate the reliability of a complex power plant with three generators, so that their future behaviour can be monitored and controlled for uninterrupted electrical power supply to the society.

Keywords
MTTF (Mean Time to Failure), R (Reliability), Q (Unreliability), P (Probability), Rg (Reliability State), Rsw (Reliability State using Weibull Distribution), Rse (Reliability State using Exponential Distribution).

1. INTRODUCTION
For several years now, deriving value from “big data” [1] has been a concern for executives focused on the distribution side of electric utilities. It was only a matter of time before generating units and fleets also had the capability to collect, analyze, and act upon huge volumes of near-real-time data [2]. In this paper, the authors’ have discussed the logical probability methods [3] of reliability analysis which describe the structure of complex equipment and the features of its functioning by the help of mathematical logic, the quantitative estimation of reliability being done with the aid of probability laws [4]. Here, the authors’ have considered a complex system having three subsystems in parallel redundancy. This complex system is identical with an engineering system consisting of three generators G1, G2 and G3 in parallel redundancy [5]. These power generators are connected with a sub power board (SPB) and some switch Boards SB1, SB2, SB3, SB4, respectively. All these components are connected with the help of perfectly reliable cables C1, C2----C9. Thus, the complex system under consideration is nothing but a multi-component power plant. The object of the system is to supply power generated by the generators G1, G2 and G3 to consumers through output board (OPB) and consequently, the reliability of the power supply is also calculated by means of Boolean function technique with considering that failure times for various components of the system follow arbitrary time distribution. Moreover an important parameter, viz., MTTF [6] has also calculated for exponential failure time distribution for different components of the system. Some numerical examples along with graphical [7] illustration have also been appended at the end to highlight the utility of the model for practical applications [8]. In the power sector, the most popular application in this category is condition monitoring and predictive maintenance of a wide variety of assets. The IoT-based approach transitions from traditional reactive [9] and periodic maintenance strategies to proactive strategies. The applications are focused on the highest value assets in generation plants, and in the transmission and distribution grid [10].

2. ASSUMPTIONS
The following assumptions have been associated with the model.
(i) The state of every component and of the whole system is either good or bad. There is no reduced efficiency state.
(ii) There is no repair facility to the failed component.
(iii) The states of all the components are statistically independent.
(iv) The reliability of every component is known in advance.
(v) The failure times of all components are arbitrary.
(vi) Initially, all the components are in working state of full efficiency.
(vii) The supply of power can fail, if at least one component in all the routes of supply fails.
(viii) The cables used, are hundred percent reliable.

3. NOTATIONS
The following notations have used throughout this model and shown in proposed model figure 1.

\[ x_1, x_2, x_3 : \text{State of generators G1, G2 and G3 respectively.} \]
\[ x_4, x_9 : \text{State of sub power board (SPB) and output board (OPB), respectively [11].} \]
\[ x_5, x_6, x_7, x_8 : \text{State of switch boards SB1, SB2, SB3 and SB4 respectively.} \]
\[ x_k = \begin{cases} 0 & \text{in failed state} \\ 1 & \text{in good state, } k = 1, 2, \ldots, 9 \end{cases} \]
**4. FORMULATION OF MATHEMATICAL MODEL**

The authors explain some of the performance measures of the system which are of interest from the system’s design point of view and analysis. In the present concerned proposed model, the performance measures of the power supply plant is quantified with respect to Reliability and Availability of the system. Where as Reliability $R(t)$ is a classical measure defined as the probability that the system functions well in the interval $(0,t)$ and is symbolically defined as $R(t) = P\{ z(u) = 1, 0 \leq u \leq t : z(0) = 1 \}$. Equivalently, let $X$ be the the random variable representing the duration of the first system failure starting with initial operable condition at $t = 0$. Then, the reliability $R(t)$ of the system is equal to $P\{ X > t \}$. and Availability $A(t)$ is defined by the probability that a system is in operating condition at time ‘$t$’. Hence, $A(t) = P\{ z(t) = 1 : z(0) = 1 \}$. As you can see the following differences between the performance measures Reliability and Availability. The reliability is an interval function while Availability is a point function describing the behavior of the system at a specified epoch. Secondly, the reliability function precludes the failure of the system during the interval under consideration, while availability function does not impose any such restriction on the behavior of the system. Associated with $A(t)$, there is another measure $A'(t) = 1 - A(t)$, probability that the system is unavailable at a time ‘$t$’. In Pointwise Availability the probability that the will be able to operate with tolerance limits at a given instant and is also called operational readiness. Symbolically, it is $A(t) = P\{ X(t) = 1 \}$ where, $X(t)$ is a binary variable having values 1 and 0, respectively for the operation and non-operation of the system at an instant ‘$t$’.

Now, the object of the proposed system is to supply power, generated by the generators $G1$, $G2$ and $G3$ through OPB to consumers. Using Boolean function technique, the authors’ have considered all possible scenarios by taking the combination of all units that are essential for a fully working power plant as the readers’ of this research paper will understand the logical matrix by seeing the first row of the matrix, for example $X_1$ is a unit resembling generator $G_1$ which is connected through cable $C_1$ to the subpower board SPB which is unit $X_4$ which is further connected through cable $C_4$ to sub power board SB1 which is unit $X_5$ and with Cable $C_7$ to the final output board OPB which is unit $X_9$. Lastly, the proposed power plant and the conditions [12] of capability of the successful operation of complex system in terms of logical matrix [13] are expressed as.

![Fig 1: The System Configuration of Proposed Model](image-url)
By application of algebra of logical, equation (1) may be written as.

\[ f(x_1, x_2, \ldots, x_9) = \left( x_4 x_9 \wedge g(x_1 x_2 x_3 x_5 x_6 x_7 x_8) \right) \quad (2) \]

Where,

\[ g(x_1 x_2 \ldots x_9) = \begin{bmatrix} A_1 & A_2 & A_3 & A_4 & A_5 & A_6 & A_7 & A_8 \end{bmatrix} \]

By using orthogonalization algorithm, equation (4) may be written as.

\[ \begin{bmatrix} A_1 & A_2 & A_3 & A_4 \end{bmatrix} \]

Using algebra of logic, one can obtain the following results.

\[ A'_i = \begin{bmatrix} x'_1 & x'_2 & \ldots & x'_{i+1} \end{bmatrix}, \quad i = 1, 2, 3, 4 \]

\[ A'_j = \begin{bmatrix} x'_1 & x'_2 & \ldots & x'_j \end{bmatrix}, \quad j = 5, 6, 7, 8 \]

\[ A'_k = \begin{bmatrix} x'_1 & x'_2 & \ldots & x'_{k-4} \end{bmatrix}, \quad k = 9, 10, 11, 12 \]

Now,

\[ A'_{1} A'_{2} = \begin{bmatrix} x'_1 & x'_2 \end{bmatrix} \wedge \begin{bmatrix} x'_1 & x'_2 & \ldots \end{bmatrix} \]

\[ = \begin{bmatrix} x'_1 & x'_2 & x'_3 \end{bmatrix} \quad (6) \]
Similarly,
\[ A_1'A_2'A_3'(x_3'x'2'x'1') \quad \text{(7)} \]
\[ A_1'A_2'A_3'A_4'A_5'(x_5'x_4'x_3'x_2'x_1') \quad \text{(8)} \]
\[ A_1'A_2'A_3'A_4'A_5'A_6'(x_6'x_5'x_4'x_3'x_2'x_1') \quad \text{(9)} \]
\[ A_1'A_2'A_3'A_4'A_5'A_6'A_7'A_8'A_9'(x_9'x_8'x_7'x_6'x_5'x_4'x_3'x_2'x_1') \quad \text{(10)} \]
\[ A_1'A_2'A_3'A_4'A_5'A_6'A_7'A_8'A_9'A_{10}A_{11} \quad \text{(11)} \]
\[ = (x_1'x_2'x_3'x_4'x_5'x_6'x_7'x_8'x_9') \quad \text{(12)} \]
\[ A_1'A_2'A_3'A_4'A_5'A_6'A_7'A_8'A_{10}A_{11}A_{12} \quad \text{(13)} \]
\[ = (x_1'x_2'x_3'x_4'x_5'x_6'x_7') \quad \text{(14)} \]
\[ A_1'A_2'A_3'A_4'A_5'A_6'A_7'A_8'A_9'A_{10}A_{11}A_{12} = (x_1'x_2'x_3'x_4'x_5'x_6'x_7') \quad \text{(15)} \]
\[ A_1'A_2'A_3'A_4'A_5'A_6'A_7'A_8'A_9'A_{10}A_{11}A_{12} = (x_1'x_2'x_3'x_4'x_5'x_6'x_7') \quad \text{(16)} \]

By using equations (6) through (16), equation (5) becomes,
\[
\begin{align*}
&g(x_1, x_2, \ldots, x_9) \\
&= x_1'x_2'x_3'x_4'x_5'x_6'x_7'
\end{align*}
\]

In view of equation (17), equation (2) becomes,
\[
\begin{align*}
&f(x_1, x_2, \ldots, x_9) \\
&= x_1'x_2'x_3'x_4'x_5'x_6'x_7'
\end{align*}
\]

Since R.H.S. of equation (18) is the disjunction of pair wise disjoint conjunctions, therefore, reliability [14] of the supply of the complex system is given by.
\[
R_s = Pr \{f(x_1, x_2, \ldots, x_9) = 1\}
\]

Where, \( R_1, R_2, \ldots, R_9 \) are the reliabilities of the components [15] of the complex system corresponding to the states \( x_1, x_2 \ldots, x_9 \) respectively.

6. SOME PARTICULAR CASES

In this segment, we are discussing the various cases in this model.

Case I: If Reliability of Every Component is Being R

Then equation (19) yields,
\[
R_s = R^9 \left[ \frac{1}{12} - 30R + 34R^2 - 21R^3 + 7R^4 - R^5 \right] \quad \text{(20)}
\]

Case II: When Failure Rates Follow Weibull Distribution

Let failure rates of generators \( G_1, G_2, G_3 \): sub power board SBP; switch boards SB1, SB2, SB3; output board OPB be \( \lambda_1, \lambda_2, \lambda_3 \) respectively, then from equation (19) reliability [16] of whole system at instant ‘t’ is given as.
\[
R_{SW}(t) = \sum_{i=1}^{23} \exp\left(-b_i t^p\right) - \sum_{j=1}^{23} \exp\left(-a_j t^p\right) = - - - - - (21)
\]

Where, \( p \) is a positive parameter and \( b_i \)'s and \( a_j \)'s are mentioned as below:
\[
\begin{align*}
b_1 &= c + \lambda_3 + \lambda_5 \\
b_2 &= c + \lambda_4 + \lambda_6 \\
b_1 &= c + \lambda_5 + \lambda_7 \\
b_6 &= c + \lambda_3 + \lambda_6 + \lambda_7 \\
b_7 &= c + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_8 &= c + \lambda_2 + \lambda_3 + \lambda_6 + \lambda_7 \\
b_9 &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 \\
b_{10} &= c + \lambda_4 + \lambda_6 \\
b_{11} &= c + \lambda_3 + \lambda_5 \\
b_{12} &= c + \lambda_2 + \lambda_4 + \lambda_6 + \lambda_7 \\
b_{13} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_6 + \lambda_7 \\
b_{14} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{15} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{16} &= c + \lambda_2 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{17} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{18} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{19} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{20} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{21} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{22} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{23} &= c + \lambda_2 + \lambda_3 + \lambda_4 + \lambda_5 + \lambda_6 + \lambda_7 \\
b_{24} &= c + \lambda_3 + \lambda_5 \\
b_{25} &= c + \lambda_3 + \lambda_6 \\
b_{26} &= c + \lambda_3 + \lambda_7 \\
\end{align*}
\]
Also,
\[
a_1 = c + \lambda_1 + \lambda_4 t + \lambda_8 \\
a_2 = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_3 = c + \lambda_1 + \lambda_4 + \lambda_7 \\
a_4 = c + \lambda_1 + \lambda_5 + \lambda_8 \\
a_5 = c + \lambda_1 + \lambda_4 + \lambda_6 \\
a_6 = c + \lambda_1 + \lambda_5 + \lambda_6 \\
a_7 = c + \lambda_1 + \lambda_4 + \lambda_5 t + \lambda_7 \\
a_8 = c + \lambda_1 + \lambda_5 t + \lambda_6 \\
a_9 = c + \lambda_1 + \lambda_5 + \lambda_7 \\
a_{10} = c + \lambda_1 + \lambda_5 + \lambda_4 t + \lambda_7 \\
a_{11} = c + \lambda_1 + \lambda_5 + \lambda_6 \\
a_{12} = c + \lambda_1 + \lambda_5 + \lambda_6 \\
a_{13} = c + \lambda_1 + \lambda_5 + \lambda_7 \\
a_{14} = c + \lambda_1 + \lambda_5 + \lambda_4 t + \lambda_7 \\
a_{15} = c + \lambda_1 + \lambda_5 + \lambda_7 \\
a_{16} = c + \lambda_1 + \lambda_5 + \lambda_7 \\
a_{17} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{18} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{19} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{20} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{21} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{22} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{23} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{24} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{25} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{26} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{27} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{28} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{29} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{30} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{31} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{32} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{33} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{34} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{35} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{36} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{37} = c + \lambda_1 + \lambda_5 t + \lambda_7 \\
a_{38} = c + \lambda_1 + \lambda_5 + \lambda_7 + \lambda_8 \\
a_{39} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{40} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{41} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{42} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{43} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{44} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{45} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{46} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{47} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{48} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{49} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{50} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{51} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\
a_{52} = c + \lambda_1 + \lambda_5 t + \lambda_7 + \lambda_8 \\

And \( c = \lambda_1 + \lambda_2 \)

**Case III: When Failure Rates Follow Exponential Time Distribution**

Exponential distribution is a particular case of Weibull distribution [17] for \( p = 1 \) and is much useful in numerous practical problems [18]. The reliability of the system in this case at any instant ‘t’, [19] can be obtained by putting \( p = 1 \) in equation [20] (21), and is

\[
R_{SW}(t) = \sum_{i=1}^{53} \exp \{-b_i t\} - \sum_{i=1}^{52} \exp \{a_j t\} \quad (22)
\]

Where, \( b_i \)'s and \( a_j \)'s are stated earlier.

\[
\mathbf{MTTF} = \int_{0}^{\infty} R_{SE}(t) \cdot dt = \sum_{i=1}^{53} \frac{1}{b} - \sum_{j=1}^{52} \frac{1}{a} \quad (23)
\]

**7. THE NUMERICAL COMPUTATION**

For the numerical illustration of the result obtained, the authors’ assume the following values of data and then we obtain the variation in the Reliability parameter with respect to Time.

Setting \( \lambda_1 = \lambda_2 = \ldots \ldots \lambda_8 = q, \) [21] we get

\[
R_{SW}(t) = 12e^{-4qt} + 30e^{-5qt} + 34e^{-6qt} - 21e^{-7qt} + 7e^{-8qt} - q^{0.05} \quad (24)
\]

\[
R_{SE}(t) = 12e^{-4qt} + 30e^{-5qt} + 34e^{-6qt} - 21e^{-7qt} + 7e^{-8qt} - q^{0.05} \quad (25)
\]

And,

\[
\mathbf{MTTF} = \frac{0.4305556}{q} \quad (26)
\]

In equation (24), putting \( p = 2, q = 0.01 \) and \( t = 0, 1, \ldots \ldots \); In equation (25), putting \( q = 0.01 \) and \( t = 0, 1, \ldots \ldots \); In equation (26), putting \( q = 0.01, 0.02 \ldots \ldots 0.1; \)
Using these \[22\] values one can compute from the tables 1 and tables 2 and sketch the graphs \[23\] as given in figure 1 and figure 2 respectively \[24\].

**Table 1: The Rsw and Rse Time Data**

<table>
<thead>
<tr>
<th>t</th>
<th>Rsw (t)</th>
<th>Rse (t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0.980198</td>
<td>0.980198</td>
</tr>
<tr>
<td>2</td>
<td>0.960782</td>
<td>0.923059</td>
</tr>
<tr>
<td>3</td>
<td>0.941740</td>
<td>0.834692</td>
</tr>
<tr>
<td>4</td>
<td>0.923059</td>
<td>0.723456</td>
</tr>
<tr>
<td>5</td>
<td>0.904727</td>
<td>0.598530</td>
</tr>
<tr>
<td>6</td>
<td>0.886735</td>
<td>0.469348</td>
</tr>
<tr>
<td>7</td>
<td>0.869071</td>
<td>0.345535</td>
</tr>
<tr>
<td>8</td>
<td>0.851729</td>
<td>0.236253</td>
</tr>
<tr>
<td>9</td>
<td>0.834692</td>
<td>0.148461</td>
</tr>
<tr>
<td>10</td>
<td>0.817958</td>
<td>0.085002</td>
</tr>
</tbody>
</table>

**Table 2: The Failure Rate and Mean Time to Failure Data**

<table>
<thead>
<tr>
<th>q</th>
<th>MTTF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.01</td>
<td>43.05556</td>
</tr>
<tr>
<td>0.02</td>
<td>21.52778</td>
</tr>
<tr>
<td>0.03</td>
<td>14.35185</td>
</tr>
<tr>
<td>0.04</td>
<td>10.76389</td>
</tr>
<tr>
<td>0.05</td>
<td>8.611112</td>
</tr>
<tr>
<td>0.06</td>
<td>7.175929</td>
</tr>
<tr>
<td>0.07</td>
<td>6.150794</td>
</tr>
<tr>
<td>0.08</td>
<td>5.381945</td>
</tr>
<tr>
<td>0.09</td>
<td>4.783951</td>
</tr>
<tr>
<td>0.10</td>
<td>4.305556</td>
</tr>
<tr>
<td>0.11</td>
<td>3.914142</td>
</tr>
</tbody>
</table>

**Fig 2: The Reliability vs Time**

**Fig 3: The Mean Time to Failure and Failure Rate**

**8. CONCLUSION**

In this paper figure 2 shows the reliability of the system at any given instant ‘t’, when failure rates follow Weibull or exponential time distribution. After making a critical examination of table and graph, the authors’ conclude that the reliability of the power plant decreases at a uniform rate when the failures rates follow exponential time distribution, but in case of Weibull distribution it decreases initially fast, but thereafter it decreases at a uniform rate. In the end figure 3 shows the graph of ‘MTTF’ versus failure rate ‘q’. MTTF decreases catastrophically in the beginning but after q = 0.05 it decreases approximately at uniform rate.

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**10. REFERENCES**


