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On Some New Signed Unidomination Parameters of Corona Product Graph of a Cycle with a Star

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ABSTRACT
Domination in graphs is an emerging area of research in graph theory and in recent years it has been studied extensively. The concept of dominating functions are introduced by Hedetniemi [6] and this concept has been studied extensively in recent years. Unidominating function and unidomination number are new concepts introduced by Anantha Lakshmi [1].She studied these concepts for some standard graphs.

A new product on two graphs $G_1$and G_2, called corona product denoted by $G_1 \odot G_2$, was introduced by Frucht and Harary [3]. The concept signed dominating function of corona product graph$C_n \odot K_m$ was studied by Siva Parvathi [8].Aruna [2] has introduced new concepts signed unidominating function and signed upper unidomination number of a graph. In this paper these concepts are studied for corona product graph$C_n \odot K_{1,m}$.

Keywords
Signed unidominating function, signed unidomination number, minimal signed unidominating function, upper signed unidomination number.

1. INTRODUCTION
Domination in graphs is an emerging area of research since few decades. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al. [4, 5].

In recent years dominating functions in domination theory is playing a key role as they have interesting applications. The concepts of dominating functions are introduced by Hedetniemi [6]. Anantha Lakshmi [1] has introduced new concepts unidomination, upper unidomination, minimal unidominating function of a graph and studied these functions for some standard graphs.

We consider corona product graphs, a new concept introduced by Frucht and Harry [3]. The corona of two graphs G and H is a graph obtained by taking one copy of G and $|V(G)|$ copies of H and then joining the i^{th} - vertex of G with all the vertices of the i^{th} - copy of H. It is denoted by $G \odot H$.

The unidominating function and upper unidomination number are new concepts introduced by Anantha Lakshmi [1] and she studied these concepts for some standard graphs. The concept signed dominating function of corona product graph$C_n \odot K_m$ was studied by Siva Parvathi [8].Aruna [2] has introduced new concepts signed unidominating function and signed upper unidomination number of a graph and she studied these concepts for some corona product graphs..

In this paper these concepts are studied for corona product graph$C_n \odot K_{1,m}$. Also the signed unidomination and upper signed unidomination number of the above said graph is found. Further, the number of signed unidominating functions of minimum weight and minimal signed unidominating functions of maximum weight for this graph are determined.

2. CORONA PRODUCT OF C_n AND$K_{1,m}$
The corona product of a cycle C_n with a complete graph $K_{1,m}$ is a graph obtained by taking one copy of a n – vertex graph C_n and n copies of $K_{1,m}$ and then joining the i^{th} - vertex of C_n to every vertex of i^{th} - copy of$K_{1,m}$. This is denoted by $C_n \odot K_{1,m}$.

The vertices in C_n are denoted byv_1,...........$v_n$and the vertices in the i^{th} copy of star graph $K_{1,m}$ are denoted by $u_{i0}, u_{i1}, u_{i2}, \dots \dots . u_{im}$respectively.

3. SIGNED UNIDOMINATING FUNCTION OF $C_n \odot K_{1,m}$
In this section the new concepts of signed unidominating function and signed unidomination number are introduced and defined as follows.

Definition 2.1: Let $G\,(V, E)$ be a graph .A function $h: V \to \{-1,1\}$ is said to be a signed unidominating function of G if

$$\sum_{u \in N[v]} h(u) \geq 1 \text{ for all } v \in V \text{and } h(v) = 1,$$

$$\sum_{u \in N[v]} h(u) = 1 \text{ for all } v \in V \text{and } h(v) = -1.$$

Definition 2.2: The signed unidomination number of a graph $G(V, E)$ is defined as min $\{h(V)/h$ is a signed unidominating function$\}$.

It is denoted by ⍰$_{su}$(G).

Here

$$h(V) = \sum_{u \in V} h(u) \text{is called as the weight of the signed unidominating function h.}$$

That is the signed unidomination number of a graph $G\,(V, E)$ is the minimum of the weights of the signed unidominating functions of G.

In what follows we determine the signed unidominating function and signed unidomination number of above corona graph.

Theorem 2.1: The signed unidomination number of the corona product graph $C_n \odot K_{1,m}$

is $\begin{cases} n & \text{if m is odd} \\ 2n & \text{if m is even.} \end{cases}$

Proof: Consider the graph $C_n \odot K_{1,m}$.

Define the function $h: V \to \{-1,1\}$ by

$h(v_i) = 1$, for $i = 1,2,3,\dots,n$,

$h(u_i) = 1$ for $i = 1,2,3,\dots\dots n$.

and

$h(u_{ij}) = \begin{cases} -1 & \text{for } \lceil \frac{m}{2} \rceil \text{ vertices in each copy of } K_{1,m} \\ 1 & \text{otherwise} \end{cases}$

for $i = 1,2,3,\dots,n$ and $j = 1,2,3,\dots,m$.

To find the signed unidomination number of $C_n \odot K_{1,m}$ the following cases arise.

Case 1: Suppose m is odd.

Then $\lceil \frac{m}{2} \rceil = \frac{m+1}{2}$.

By the definition of the function -1 is assigned to $\frac{m+1}{2}$ vertices in each copy of $K_{1,m}$ and 1 is assigned to $\frac{m-1}{2}$ vertices in each copy of $K_{1,m}$ in $C_n \odot K_{1,m}$.

Now we verify that h is a signed unidominating function.

If $v_i \in C_n$, then

$$\sum_{u \in N[v_i]} h(u) = h(v_{i-1}) + h(v_i) + h(v_{i+1}) + h(u_i) + h(u_{i1}) + \dots + h(u_{im})$$

$$= 1 + 1 + 1 + 1 + [\frac{m+1}{2}(-1) + \frac{m-1}{2}(1)]$$

$$= 4 + \left[-\frac{m}{2} - \frac{1}{2} + \frac{m}{2} - \frac{1}{2}\right] = 4 - 1 = 3 .$$

If $u_i \in K_{1,m}$, then

$$\sum_{u \in N[u_i]} h(u_i) = h(v_i) + h(u_i) + h(u_{i1}) + h(u_{i2}) + \dots + h(u_{im})$$

$$= 1 + 1 + [\frac{m+1}{2}(-1) + \frac{m-1}{2}(1)]$$

$$= 2 + \left[-\frac{m}{2} - \frac{1}{2} + \frac{m}{2} - \frac{1}{2}\right]$$

$$= 2 - 1 = 1.$$

If $u_{ij} \in K_{1,m}$, then $h(u_{ij}) = -1$ and $h(u_{ij}) = 1$.

Let $u_{ij} \in K_{1,m}$ and $h(u_{ij}) = -1$. Then

$$\sum_{u \in N[u_{ij}]} h(u) = h(v_i) + h(u_i) + h(u_{ij})$$

$$= 1 + 1 + (-1) = 1.$$

Let $u_{ij} \in K_{1,m}$ and $h(u_{ij}) = 1$. Then

$$\sum_{u \in N[u_{ij}]} h(u) = h(v_i) + h(u_i) + h(u_{ij})$$

$$= 1 + 1 + 1 = 3.$$

That is h satisfying the conditions of a signed unidominating function and hence it follows that h is a signed unidominating function.

$$\text{Now} h(V) = \sum_{u \in V} h(u) = \sum_{u \in C_n} h(u) + \sum_{u \in K_{1,m}} h(u)$$

$$= \underbrace{(1 + 1 + \dots + 1)}_{n-times} + \{\underbrace{\left[1 + \frac{m+1}{2}(-1) + \frac{m-1}{2}(1)\right] + .. + [1 + \frac{m+1}{2}(-1) + \frac{m-1}{2}(1)]}_{n-times}\}$$

$$= n + 0 = n.$$

Thus $h(V) = n$.

Now for all other possibilities of assigning values 1 and -1 to the vertices of C_n and vertex u_i and vertices u_{ij} in each copy of $K_{1,m}$, we can show that the resulting functions are not signed unidominating functions.

Therefore the function defined above is the only signed unidominating function.

Therefore $⍰_{su}(C_n \odot K_{1,m}) = n$ when m is odd.

Case 2: Suppose m is even.

Then $\lceil \frac{m}{2} \rceil = \frac{m}{2}$.

By the definition of the function -1 is assigned to $\frac{m}{2}$ vertices in each copy of $K_{1,m}$ and 1 is assigned to $\frac{m}{2}$ vertices in each copy of $K_{1,m}$ in $C_n \odot K_{1,m}$.

If $v_i \in C_n$, then

$$\sum_{u \in N[v_i]} h(u) = h(v_{i-1}) + h(v_i) + h(v_{i+1}) + h(u_i) + h(u_{i1}) + \dots + h(u_{im})$$

$$= 1 + 1 + 1 + 1 + [\frac{m}{2}(-1) + \frac{m}{2}(1)]$$

$$= 4.$$

If $u_i \in K_{1,m}$, then

$$\sum_{u \in N[v_i]} h(u_i) = h(v_i) + h(u_i) + h(u_{i1}) + h(u_{i2}) + \dots + h(u_{im})$$

$$= 1 + 1 + \left[\frac{m}{2}(-1) + \frac{m}{2}(1)\right] = 2.$$

If $u_{ij} \in K_{1,m}$, then $h(u_{ij}) = -1$ and $h(u_{ij}) = 1$

Let $u_{ij} \in K_{1,m}$ and $h(u_{ij}) = -1$. Then

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$$\sum_{u\in N[u_{ij}]} h(u) = h(v_i) + h(u_i) + h(u_{ij})$$

$$= -1 + 1 + 1 = 1.$$

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If $u_{ij} \in K_{1,m}$ and $h(u_{ij}) = 1$. Then

$$\sum_{u\in N[u_{ij}]} h(u) = h(v_i) + h(u_i) + h(u_{ij})$$

$$= 1 + 1 + 1 = 3.$$

Hence it follows that h is a signed unidominating function.

Now $h(V) = \sum_{u\in V} h(u) = \sum_{u\in C_n} h(u) + \sum_{u\in K_{1,m}} h(u)$

$$= \underbrace{(1 + 1 + \cdots + 1)}_{n-\text{times}} + \underbrace{\{[1 + \tfrac{m}{2}(-1) + \tfrac{m}{2}(1)] + \cdots + [1 + \tfrac{m}{2}(-1) + \tfrac{m}{2}(1)]\}}_{n-\text{times}}$$

$= n + n = 2n.$

Thus $h(V) = 2n$.

Now for all other possibilities of assigning values 1 and -1 to the vertices of C_n and vertex u_i and vertices u_{ij} in each copy of $K_{1,m}$, we can show that the resulting functions are not signed unidominating functions.

Therefore the function defined above is the only signed unidominating function.

Therefore $?_{su}(C_n \odot K_{1,m}) = 2n$ when m is even.

Theorem 2.2: If m is odd then the number of signed unidominating functions of $C_n \odot K_{1,m}$ with minimum weight n is 1 and if m is even then the number of signed unidominating functions of $C_n \odot K_{1,m}$ with minimum weight 2n is 1.

Proof: Follows by Theorem 2.1.

4. UPPER SIGNED UNIDOMINATION NUMBER OF $C_n \odot K_{1,m}$

In this section the new concepts of minimal signed unidominating function and upper signed unidomination number are introduced and defined as follows.

Definition 3.1: Let $G(V, E)$ be a connected graph and h, g be functions from V to$\{-1,1\}$. We say that $h < g$ if $h(u) \le g(u)\ \forall\, u \in V$, with strict inequality for atleast one vertex u.

Definition 3.2: Let $G(V, E)$ be a connected graph. A signed unidominating function $h: V \to \{-1,1\}$ is called a minimal signed unidominating function if for all $g < h$, g is not a signed unidominating function.

Definition 3.3: The upper signed unidomination number of a graph $G(V, E)$ is defined as max $\{h(V)$ /h is a minimal signed unidominating function$\}$.

It is denoted by $\Gamma_{su}(G)$.

Theorem 3.1: The upper signed unidomination number of corona product graph $C_n \odot K_{1,m}$

is$\begin{cases} n \text{ if m is odd} \\ 2n \text{ if m is even.}\end{cases}$

Proof: Consider the graph $C_n \odot K_{1,m}$.

Define a function $f: V \to \{-1,1\}$ by

$h(v_i) = 1,\ v_i \in C_n$ for $i = 1,2,3, \ldots \ldots n,$

$hu_i) = 1,\ u_i \in K_{1,m}$ for $i = 1,2,3, \ldots \ldots n,$

and

$$h(u_{ij}) = \begin{cases} -1 & \text{for } \left\lceil \frac{m}{2} \right\rceil \text{ vertices in each copy of } K_{1,m}, \\ 1 & \text{otherwise.} \end{cases}$$

This function is same as the function defined in Theorem 2.1 and it is shown that h is a signed unidominating function.

Now we check for the minimality of h.

Define a function $g: V \to \{-1,1\}$ by

$$g(v) = \begin{cases} -1, & \text{for one vertex } v = v_k \text{ of} C_n \text{ in G}, \\ -1 & \text{for } \left\lceil \frac{m}{2} \right\rceil \text{ vertices in each copy of } K_{1,m}, \\ 1, & \text{oterwise.} \end{cases}$$

Then by the definition of g it is obvious that $g < h$.

To find the upper signed unidominationnumber of $C_n \odot K_{1,m}$ the following cases arise.

Case 1: Suppose m is odd.

Then $\left\lceil \frac{m}{2} \right\rceil = \frac{m+1}{2}$.

By the definition of the function -1 is assigned to $\frac{m+1}{2}$vertices and 1 is assigned to $\frac{m-1}{2}$ vertices in each copy of $K_{1,m}$ in $C_n \Theta K_{1,m}$.

Let $i = k$.

If $u_k \in k_{1,m}$, then

$$\sum_{u\in N[u_k]} g(u) = g(u_k) + g(v_k) + g(u_{k1}) + \ldots\ldots.. + g(u_{km})$$

$$= 1 + (-1) + \left[\frac{m+1}{2}(-1) + \frac{m-1}{2}(1)\right]$$

$$= 0 + \left[-\frac{m}{2} - \frac{1}{2} + \frac{m}{2} - \frac{1}{2}\right]$$

$$= 0 - 1 = -1.$$

That is the condition for signed unidominating function fails in the neighbourhood of the vertex $u_k \in k_{1,m}$.

Thus g is not a signed unidominating function.

Since g is defined arbitrarily, it follows that there exists no $g < h$ such that g is a signed unidominating function.

Hence h is a minimal signed unidominating function.

Further h is the only one minimal signed unidominating function because any other possible assignment of values $-1, 1$ to the vertices of C_n and $K_{1,m}$ does not make h no more a signed unidominating function.

Now $h(V) = \sum_{u \in V} h(u) = \sum_{u \in C_n} h(u) + \sum_{u \in K_{1,m}} h(u)$

$$= \underbrace{(1 + 1 + \cdots + 1)}_{n-times} + \underbrace{\{[1 + \frac{m+1}{2}(-1) + \frac{m-1}{2}(1) + \cdots + [1 + \frac{m+1}{2}(-1) + \frac{m-1}{2}(1)\}}_{n-times}$$

$$= n + 0 = n.$$

Thus $h(V) = n$.

Now $\max\{h(V)$ /h is a minimal signed unidominating function$\} = n$, because h is the only one minimal signed unidominating function.

Therefore $\Gamma_{su}(C_n \odot K_{1,m}) = n$ when m is odd.

Case 2: Suppose m is even.

Then $\left\lceil \frac{m}{2} \right\rceil = \frac{m}{2}$.

By the definition of the function -1 is assigned to $\frac{m}{2}$ vertices and 1 is assigned to $\frac{m}{2}$ vertices in each copy of $K_{1,m}$ in $C_n \odot K_{1,m}$.

Let $i = k$.

If $u_k \in k_{1,m}$, then

$$\sum_{u \in N[u_k]} g(u) = g(u_k) + g(v_k) + g(u_{k1}) + \ldots\ldots + g(u_{km})$$

$$= 1 + (-1) + \left[\frac{m}{2}(-1) + \frac{m}{2}(1)\right] = 0.$$

That is the condition for signed unidominating function fails in the neighbourhood of the vertex $u_i \in k_{1,m}$.

Thus g is not a signed unidominating function.

Since g is defined arbitrarily, it follows that there exists no $g < h$ such that g is a signed unidominating function.

Hence h is a minimal signed unidominating function.

Further h is the only one minimal signed unidominating function because any other possible assignment of values $-1, 1$ to the vertices of C_n and $K_{1,m}$ does not make h no more a signed unidominating function.

Now $h(V) = \sum_{u \in V} h(u) = \sum_{u \in C_n} h(u) + \sum_{u \in K_{1,m}} h(u)$

$$= \underbrace{(1 + 1 + \cdots + 1)}_{n-times} + \underbrace{\{[1 + \frac{m}{2}(-1) + \frac{m}{2}(1)] + \cdots + [1 + \frac{m}{2}(-1) + \frac{m}{2}(1)]\}}_{n-times}$$

$$= n + n = 2n.$$

Thus $h(V) = 2n$.

Now $\max\{h(V)$ /h is a minimal signed unidominating function$\} = 2n$, because h is the only one minimal signed unidominating function.

Therefore $\Gamma_{su}(C_n \odot K_{1,m}) = 2n$ when m is even.

Theorem 3.2: If m is odd then the number of minimal signed unidominating functions of $C_n \odot K_{1,m}$ with maximum weight n is 1 and if m is even then the number of minimal signed unidominating functions of $C_n \odot K_{1,m}$ with maximum weight n is 1.

Proof: Follows by Theorem 3.1

5. ILLUSTRATION

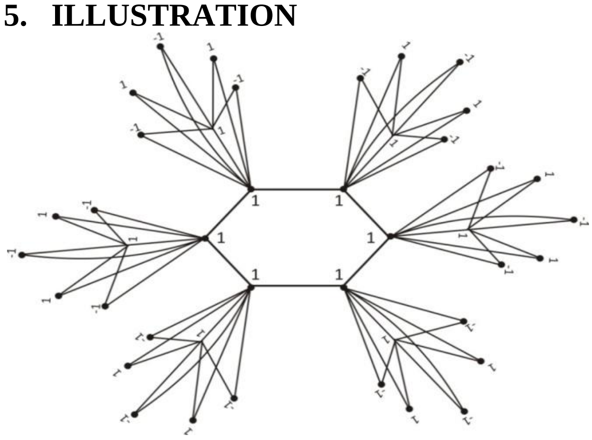

Fig. 1 $C_6 \odot K_{1,5}$

Signed unidomination number and upper signed unidomination number of the corona product graph $C_6 \odot K_{1,5}$ is 6.

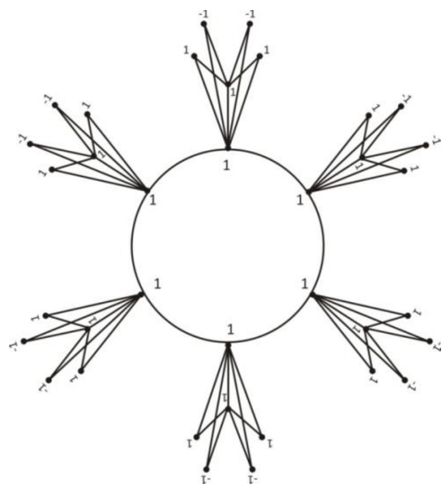

Fig. 2 $C_6 \odot K_{1,4}$

signed unidomination number and upper signed unidomination number of the corona product graph $C_6 \odot K_{1,4}$ is 12.

6. CONCLUSION

The study of corona product of some standard graphs is interesting and it gives scope for further investigations on these graphs. Total unidomination number and upper total unidomination number of corona product graph of a wheel with a star is studied by Hemalatha [7]. Finding signed unidomination and upper signed unidomination number of corona product graph of a cycle with a star throws light on further study of corona product graphs of some other standard graphs such as cycle with a path, cycle with a wheel and etc.

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