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Euler and Quaternion Parameterization in VTOL UAV Dynamics with Test Model Efficiency

Redouane Dargham
Doctoral Student in Computer Engineering National Higher School of Electricity and Mechanics
(EAS, LISER, ENSEM)

Adil Sayouti
PhD Professor,
National Higher School of Electricity and Mechanics
(ENSEM, EAS, LISER)

Hicham Medromi
PhD ,Professor and Director
National Higher School of Electricity and Mechanics
(ENSEM, EAS, LISER)

ABSTRACT
The Vetical Take Off And Landing Unmanned Aerial Vehicle (VTOL UAV) has seen unprecedented levels of growth over the past 20 years with military applications dominating the field and the civilian applications tending to follow. Further, the use of UAVs has become a favored solution for important tasks requiring air operations as aerial photography, surveillance, inspection, search and rescue or mapping .For these applications to emerge, motion control algorithms that guarantee a good amount of stability and robustness against state measurement/estimation errors are needed. This paper describes: The dynamical equation of rigid bodies can be gathered from the classical Newton-Euler differential equations, which commonly make use of the Euler angles parametrization and its limits after the description of motion in terms of quaternions formulation instead of the Euler one and its benifits. This kind of analysis, proved by some numerical results presented, has a great importance due to the applicability of quaternion to drone control. An illustration of this study will be given in an application of control of an autonomous hexacopter developed by the team architecture of systems, in the national engineering school of electricity and mechanic in Casablanca in Morocco.

Keywords
Vertical Take Off And Landing Unmanned Aerial Vehicle (VTOL UAV), motion control, Newton-Euler equations, Quaternion, Hexacopter

1. INTRODUCTION
The growing interest for the flying robots research community is partly due to the numerous applications that can be addressed with such systems like surveillance, inspection, or mapping. Recent technological advances in sensors, batteries and processing cards, allow embarking on small vehicles all components necessary for autonomous flights at a reasonable cost, but constituting also a favorable factor to help raising several issues particularly how to fly nicely without slamming into obstacles and without having weird oscillations. The first order of business to overcome in most of these links is focusing to find out an interesting way to study the motion control and the stability of those aerial robots. This requires efficient model that can be used for designing a robust controller.

Among the multicopter typology, the four rotors, also called quadrotor, have been widely chosen by many researchers as a very promising vehicle for indoor and outdoor navigation. Nowadays, the design of multicopter with more than four rotors, i.e. hexacopter and octocopter, is offring the possibility of managing one or more engine failures and to increase the total payload. In this paper a hexacopter is considered whose six-rotors are located on vertices of a hexagon and are equidistant from the center of gravity; moreover, the propulsion system consists of three pairs of counter-rotating fixed-pitch blades (Fig.1).

Fig.1.ENSEM hexacopter

The aircraft dynamic behavior is here presented by the mathematical model, by considering all its external and internal influences. Assuming the hexacopter as a rigid body, the differential equations describing its dynamic behavior can be derived from the Newton-Euler equations, leading to equivalent mathematical models. Euler angle parameterization of three-dimensional rotations contains singular points in the coordinate space that can cause failure of both dynamical models and control. These singularities are not present if the three-dimensional rotations are parameterized in terms of quaternion.

In this optic the outline of this paper is as follows: Part 2 concerns the mathematical model of the hexacopter in term of Newton-Euler equations and its limitation then in part 3, the quaternions parameterization and its benefits are introduced; finally, the Matlab simulation test for model efficiency is shown in part 4.

2. THE MATHEMATICAL MODEL NEWTON-EULER EQUATIONS
Two coordinate systems are needed to describe the motion of a hexarotor: an earth fixed frame E and a body fixed frame B. This section deals with the coordinate systems and the reference frames chosen to describe the hexacopter dynamics.

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First of all, the classical Euler parameterization is treated; in other words, the angular orientation of the aircraft's body is described by three Euler angels that represent an ordered set of sequential rotations from a reference frame to the body a frame.

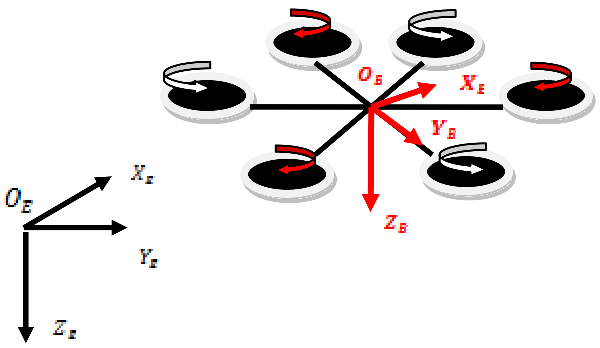

Fig. 2: The two coordinate systems that are used to describe the hexarotor's motions

The earth fixed frame, *E* ,uses NED (North, East, Down) coordinates. Its origin, denoted $\boldsymbol{O_E}$, is fixed on the earth's surface in the hexarotor's starting position. Since the earth fixed frame acts as an inertial frame the hexarotor's absolute linear position can be defined in this frame. The axes are denoted *XE*, *YE* and *ZE* respectively and their directions are shown in Figure.2.The body fixed coordinate system, *B*, is fixed in the centre of the hexarotor's airframe and its origin is $\boldsymbol{O_B}$. Consequently, the body fixed frame moves relative to the earth fixed frame when the hexarotor moves. The body fixed *x*-axis points in the forward direction, the *y*-axis points to the right and the *z*-axis points downwards. These axes are denoted X_B, Y_B and $Z_B$respectively [1].

The position of the body fixed frame in the earth fixed frame is defined as $\xi = (x \quad y \quad z)^T$ and the body frame's orientation in relation to the earth frame is described by the vector $\eta = (\emptyset \ \Theta \ \psi)^T$ where the angles $\emptyset$, Θ and ψ are called roll, pitch and yaw respectively. These angles are the Euler rotation angles.

The linear velocities of the hexarotor in the body frame is defined as $V_B = (u, v, w)^T$ and its angular velocities are defined as $\omega = (p, q, r)^T$.

In flight mechanics, the Euler angles are often used for transformations between coordinate systems. These transformations are achieved with rotation matrices which consist of terms of Euler angles. A multiplication of a rotation matrix and a vector in one coordinate system transforms that vector to another coordinate system. This part will present the rotation matrices that transform linear quantities between the two coordinate systems *E* and *B*.

Consider that the hexarotor has changed its roll, pitch and yaw angle in relation to the earth fixed frame. To describe this rotation each angle rotation is treated successively. $R_{EB} = R(\emptyset)R(\theta)R(\psi)$ is the rotation matrix that transforms a linear quantity from earth fixed coordinates to body fixed coordinates[2], [3], [4].

$$\begin{pmatrix} X_B \\ Y_B \\ Z_B \end{pmatrix} = R_{EB} \begin{pmatrix} X_E \\ Y_E \\ Z_E \end{pmatrix} =$$

$$= \begin{pmatrix} c\theta c\psi & c\theta s\psi & -S\theta \\ s\emptyset s\theta c\psi - c\emptyset s\psi & s\emptyset s\theta s\psi + c\emptyset c\psi & s\emptyset c\theta \\ c\emptyset s\theta c\psi + s\emptyset s\psi & c\emptyset s\theta s\psi - s\emptyset c\psi & c\emptyset c\psi \end{pmatrix} \begin{pmatrix} X_E \\ Y_E \\ Z_E \end{pmatrix}$$

The inverse of R_{EB} yields the rotation matrix R_{BE} that transforms linear quantities from body fixed coordinates to earth fixed coordinates (Note that the notation *c* is used for *cos*, *s* for *sin* and *t* for *tan)*. Since R_{EB} is orthogonal its inverse is given by its transpose, which gives:

$$R_{BE} = R_{EB=}^{-1}$$

$$= R_{EB}^T \begin{pmatrix} c\theta c\psi & s\theta s\theta c\psi - c\emptyset s\psi & c\emptyset s\theta s\psi - s\emptyset c\psi \\ c\theta s\psi & s\emptyset s\theta s\psi + c\emptyset c\psi & c\emptyset s\theta s\psi - s\emptyset c\psi \\ -S\theta & s\emptyset c\theta & c\emptyset c\psi \end{pmatrix} \begin{pmatrix} X_E \\ Y_E \\ Z_E \end{pmatrix}$$

The translational velocities of the body frame are linear. The transformation of these velocities from the body frame to the earth frame is thus described by:

$$\dot{\xi} = R_{BE} V_B$$

The transfer matrix that transforms the time derivatives of the Euler angles in the earth fixed frame to the angular velocities in the body fixed frame that ensure the relationship between the angular of velocity ω_B and η_E is given:

$$\omega_B = \begin{pmatrix} \dot{\emptyset} - \dot{\psi} s\theta \\ \dot{\theta} c\emptyset + \dot{\psi} s\emptyset c\theta \\ -\dot{\theta} s\theta + \dot{\psi} c\emptyset c\theta \end{pmatrix} = R_{EB}\, \dot{\eta}_E$$

Where :

$$R_{EB} = \begin{pmatrix} 1 & 0 & -s\theta \\ 0 & c\emptyset & s\emptyset c\theta \\ 0 & -s\emptyset & c\emptyset c\theta \end{pmatrix}$$

Inversion of R_{EB} gives $\boldsymbol{R_{BE}}$ **:**

$$R_{BE} = \begin{pmatrix} 1 & s\emptyset t\theta & c\emptyset t\theta \\ 0 & c\emptyset & -s\theta \\ 0 & \frac{s\emptyset}{c\theta} & \frac{c\emptyset}{c\theta} \end{pmatrix}$$

which relates $\boldsymbol{\omega_B}$ and $\dot{\boldsymbol{\eta}}_E$ as :

$$\dot{\eta}_E = R_{BE}\omega_B$$

It is important to observe that $\boldsymbol{R_{BE}}$ can be defined if and only if $\boldsymbol{\Theta} \neq \pm \frac{\pi}{2} + k\pi$, $(k \in Z)$ the main effect of Euler formulation that leads to the gimbal lock, typical situation in which a degree of freedom is lost. To solve this problem, it is possible to consider a different representation for the hexacopter orientation in space. The aircraft rotation from one frame of reference to another will be identified by four parameters, known as quaternions, whose general structure is briefly summarized afterwards. The advantages of an approach based on quaternions consist not only in the absence of singularities but also in the simplicity of computation.

3. THE QUATERNIONS MODEL

In the above section, the Euler angles can be available representation for the rotation of a rigid body in space; however, the problem of singularity leads to adopt a new parameterization, the quaternions, with the purpose to

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describe the orientation of the UAV with respect to the earth fixed frame . The quaternions [5], [6] were first proposed by Hamilton in 1843 and further studied by Cayley and Klein. A unit quaternion has the form:

$$\boldsymbol{q = q_0 + q_1 i + q_2 j + q_3 k = (q_0\ q_1\ q_2\ q_3)^T}$$

Where q_0,q_1,q_2 q_3 are real numbers satisfying $q_0^2 + q_1^2 + q_2^2 + q_3^2 = 1$ and called constituents of the quaternion *q*, and *i*, *j*, *k* are imaginary units that satisfy

$$\boldsymbol{i^2 = j^2 = k^2 = -1}$$

with $\boldsymbol{ij = -ji = k, jk = -kj = i, ki = -ik = j}$

The quaternions had already proved their efficiency in several applications, such as computer game development and 3D virtual worlds, but also as a method for rigid body rotation in three-dimensional space. The quaternion representation is based on the Euler's rotation theorem which states that any rigid body displacement where a point is fixed is equivalent to a rotation: if $\boldsymbol{\alpha}$ is the rotation angle about the unit vector $\mathbf{u} = (\boldsymbol{u_1\ u_2\ u_3})$, it is possible to define a quaternion as $\boldsymbol{q} =$ ***q0 q1 q2 q3T***,with ***q0= cos(α/2), q1= sinα2u1, q2=*** $\boldsymbol{sin\left(\frac{\alpha}{2}\right)u_2}$ and $\boldsymbol{q_3 = sin\left(\frac{\alpha}{2}\right)u_3}$ [7]. Unlike Euler angles, quaternion rotations don't require a set of predefined rotation axes because they can change its single axis continuously. Due to the fact that the method of rotating around an arbitrary direction has only one axis of rotation, degrees of freedom can't be lost; therefore gimbal lock can't occur. Body rotation in the earth frame can be represented with quaternions for each rotation about each axis. There is a connexion between the quaternion and the Euler angles.It can be shown with the following expression [8]:

$$\boldsymbol{q = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} = \begin{pmatrix} c\left(\frac{\emptyset}{2}\right)c\left(\frac{\theta}{2}\right)c\left(\frac{\psi}{2}\right) + s\left(\frac{\emptyset}{2}\right)s\left(\frac{\theta}{2}\right)s\left(\frac{\psi}{2}\right) \\ s\left(\frac{\emptyset}{2}\right)c\left(\frac{\theta}{2}\right)c\left(\frac{\psi}{2}\right) - c\left(\frac{\emptyset}{2}\right)s\left(\frac{\theta}{2}\right)s\left(\frac{\psi}{2}\right) \\ c\left(\frac{\emptyset}{2}\right)s\left(\frac{\theta}{2}\right)c\left(\frac{\psi}{2}\right) + s\left(\frac{\emptyset}{2}\right)c\left(\frac{\theta}{2}\right)s\left(\frac{\psi}{2}\right) \\ c\left(\frac{\emptyset}{2}\right)c\left(\frac{\theta}{2}\right)s\left(\frac{\psi}{2}\right) - s\left(\frac{\emptyset}{2}\right)s\left(\frac{\theta}{2}\right)c\left(\frac{\psi}{2}\right) \end{pmatrix}}$$

Similarly, a conversion from quaterions to Euler angles is given by:

$$\boldsymbol{\begin{pmatrix} \emptyset \\ \theta \\ \psi \end{pmatrix} = \begin{pmatrix} arctan\left(\frac{2(q_2q_3 + q_0q_1)}{2q_0^2 + 2q_3^2 - 1}\right) \\ arcsin\left(2(q_0q_2 - q_1q_3)\right) \\ arctan\frac{2(q_1q_2 + q_0q_3)}{2q_0^2 + 2q_1^2 - 1} \end{pmatrix}}$$

The transformation from the body reference frame *B* to the earth reference frame *E* is done with the matrix:

$$\boldsymbol{Q_{BE} = \begin{pmatrix} 2q_0^2 + 2q_1^2 - 1 & 2(q_1q_2 - q_0q_3) & 2(q_1q_3 + q_0q_2) \\ 2(q_1q_2 + q_0q_3) & 2q_0^2 + q_2^2 - 1 & 2(q_2q_3 - q_0q_1) \\ 2(q_1q_3 - q_0q_2) & 2(q_2q_3 + q_0q_1) & 2q_0^2 + q_3^2 - 1 \end{pmatrix}}$$

and from earth reference frame *E* to body reference frame *B* as:

$$\boldsymbol{Q_{EB} = {Q_{BE}}^T = \begin{pmatrix} 2q_0^2 + 2q_1^2 - 1 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & 2q_0^2 + q_2^2 - 1 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & 2q_0^2 + q_3^2 - 1 \end{pmatrix}}$$

The transformation of the translational velocities representation from the body frame to the inertial one can be expressed by $\boldsymbol{V_E = Q_{BE}V_B}$.

As the matrix *R*, Q_{BE} is orthogonal; therefore, it is $Q_{BE}^{-1} = Q^T$.

As the angular velocities concerns, the involved transformation can be written as $\dot{\boldsymbol{q}} = \boldsymbol{S\omega}$ where the matrix $\boldsymbol{S}$ depends on quaternion components as follows:

$$\boldsymbol{S = \frac{1}{2}\begin{pmatrix} -q_1 & -q_2 & -q_3 \\ q_0 & -q_3 & q_2 \\ q_3 & q_0 & -q_1 \\ -q_2 & q_1 & q_0 \end{pmatrix}}$$

On the other hand, it is possible to consider the transformation matrix depending on the angular velocity components, obtaining the link between quaternions and their derivatives with respect to time, that are:

$$\boldsymbol{\dot{q} = \begin{pmatrix} \dot{q_0} \\ \dot{q_1} \\ \dot{q_2} \\ \dot{q_3} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{pmatrix} \cdot \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix}}$$

The quaternion approach is fully equivalent to the Euler one, but it is more easy and efficient from a computational point of view and it does not exhibit the gimbal lock issues.

The aim consists in describing the motion of hexacopter, with three pairs of counter-rotating propellers arranged on vertices the hexagon. Supposed the drone as a rigid body, its dynamics is deduced from the classical Newton - Euler equations but in terms of quaternions as shown in [9]. Taking into account all the internal and external influences, the translational and rotational components of the motion read:

$$\boldsymbol{m\ddot{\xi} = F_g + Q_{BE}T_B}$$

$$\boldsymbol{\ddot{q} = \frac{d}{dt}(S\omega)}$$

in which m is the mass of the drone, $\boldsymbol{\xi} = (\boldsymbol{x, y, z})^{\boldsymbol{T}}$ represents its position vector with respect to the inertial frame, $\boldsymbol{F_g}$ is the gravitational force, $\boldsymbol{T_B}$ is the total thrust, $\boldsymbol{Q_{BE}}$ is the orthogonal transformation matrix from the body frame to the inertial one, $\boldsymbol{S}$ is the velocity transformation matrix and $\boldsymbol{\omega = (p, q, r)^T}$ is the angular velocity solution of

$$\boldsymbol{I\dot{\omega} + \omega \times (I\,\omega) + \mathcal{T} = \tau_B}$$

where *I* is diagonal inertial matrix, $\boldsymbol{\mathcal{T}}$ represents the gyroscopic effects and $\boldsymbol{\tau_{B=}(\tau_\emptyset \tau_\theta \tau_\psi)^T}$ the roll, pitch and yaw moment torque vector. Given an initial condition, the mathematical model is sufficient to describe the evolution of the aircraft, but it does not give information on the final position of the drone. It is therefore essential to associate with the mathematical model a control technique enabling to maneuver the drone, to manage the flight and to decide in advance which position will occupy the aircraft.

4. SIMULATION TEST FOR MODEL EFFICIENCY

A lot of experimental testing has been performed to validate the efficiency of the quaternion parameterization and to proof that the quaternions [10] are computationally more efficient, simple to compute with both linearity and lack of singularities characteristics. The purpose of these features is reflected in a gain of time that is significant in real flight simulations. Fig. 2 shows the execution time versus the number of points in which the temporal interval is divided: dashed line is related to the Euler formulation, while the continuous line to the quaternions formulation. It is evident the gain of time with increasing the number of points, despite the system counts two more equations with respect to the system using the Euler parametrization.

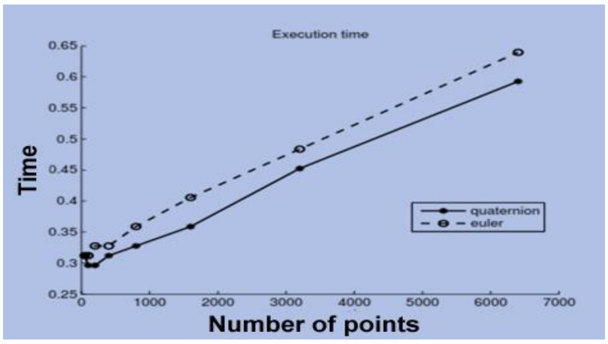

Fig. 2. Comparison of execution time between Euler (dashed) and quaternion (continuous) formulation

Moreover, while Euler formulation suffers from the gimbal lock, quaternion parametrization does not encounter it: as shown in Fig. 3, when pitch angle is close to θ, roll and yaw angles exhibit a jump.

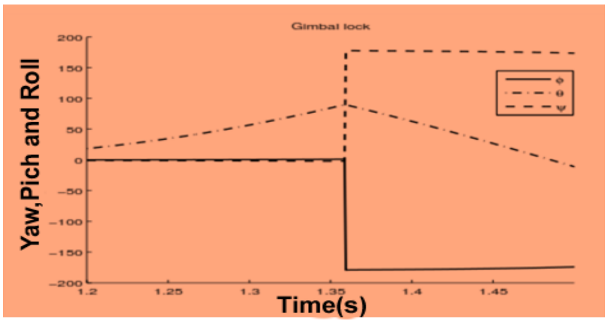

Fig.3. Evolution of Euler angles

In Figure 4 quaternion component are continuous and smooth in time.

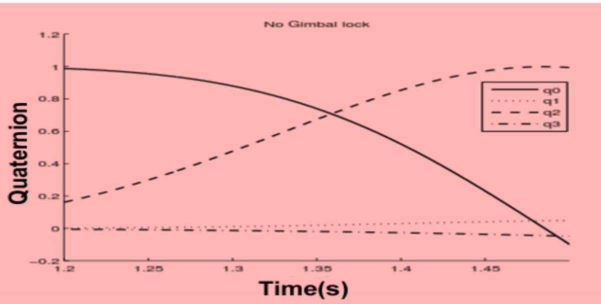

Fig.4. Evolution of quaternion

5. CONCLUSION

In this work a comparison between Euler and quaternion approach has been driven, highlighting the efficiency of the second method from a computational point of view. This application will be used in describing and controlling a small UAV (Hexacopter). The small computational time with free gimbal lock error in integration provide good answers in real flight simulations for an easier drone management and maneuverability.

The advantage in considering the quaternion reference is twofold because it avoids critical positions and, it offers a model with the linearity of the coefficients of the transformation matrix, it is also numerically more efficient and stable compared to traditional rotation formulation.

By the way the quaternion parameterization is taken into account because of its simplicity for computation and its numerical stability which allows more efficient and fast algorithm implementation. In this vision a .real applications using the quaternions with good control system will be implemented in an autonomous hexacopter developed by Team of architecture and systems in the national engineering school of electricity and mechanic (ENSEM) as practice result for concrete use.

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